


2001

Generation planning for electric power utilities under market uncertainties: a real options approach

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Generation planning for electric power utilities under market uncertainties:

A real options approach

by

Chung-Hsiao Wang

A dissertation submitted to the graduate faculty
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Industrial Engineering

Major Professor: K. Jo Min

Iowa State University

Ames, Iowa

2001

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ACKNOWLEDGEMENTS

First of all, I would like to express my sincere appreciation and thanks to my major professor, Dr. K. Jo Min, for his advice, guidance, and patience through all phases of my graduate study and the preparation of this dissertation.

I would also like to thank Dr. S. Keith Adams, Dr. Tim Van Voorhis, Dr. Janet Peterson, and Dr. W. Robert Stephenson for their helpful comments, feedbacks, and suggestions regarding my research as well as their participation on my committee. I am also grateful to Dr. Ananda Weerasinghe for his helpful comments.

I would like to thank Dr. K. Jo Min, ISU Electric Power Research Center, MidAmerican Energy, and Electric Power Research Institute (EPRI) for their financial supports.

My warmest thanks go to my mom, Show-Yueh Lin, my dad, Chih-Chun Wang, my sister, Hong-Hsiao Wang, and my fiancée, Li-Ming Liu. Without their endless love, understanding, and encouragement, I could not have finished my graduate study.

This dissertation is dedicated to my mom, who passed away on May 27, 1998.

ABSTRACT

The generation business in the U.S. is currently undergoing a transition from a regulated monopoly toward an uncertain, competitive market. Under the competitive market, the price of electric power as well as the corresponding revenue may be much less certain than before. These market uncertainties have increased the significance of two critical factors in generation planning. These factors are financial risks and managerial flexibilities.

In order to quantitatively and objectively address these two factors in generation planning, in this dissertation, we design and analyze a series of mathematical models based on the real options approach for generation planning. Hence, this dissertation can be viewed as a comprehensive study of the real options approach in generation planning.

The dissertation begins with a simple multiple-project single-option model based on the Black-Scholes option-pricing formula. This is followed by a single-project multiple-option model based on geometric Brownian motion process, binomial lattice, and backward dynamic programming.

Next, we design and analyze sophisticated multiple-project multiple-option models where the market values of the projects are assumed to be correlated. As before, we employ the backward dynamic programming over the lattice to determine the optimal options for the multiple projects and the corresponding values of the investment. Also, we investigate the roles of the correlation coefficients among projects in decision making and the value of an option.

In addition, we construct and analyze a traditional generation planning model that incorporates forced customer outage costs and forced utility outage costs. By incorporating

forced customer outage costs, we attempt to take customer satisfaction level into account.

We compare and contrast the models from the real options approach as well as the traditional approach.

We hope that the results of this dissertation will encourage utilities to effectively utilize the real options approach in generation planning under market uncertainties. As this approach can address the financial risks and managerial flexibility while the classical discounted cash flow approaches can not, we also hope that generation planning can be performed more quantitatively and objectively under the new economic uncertainties.

CHAPTER 1. INTRODUCTION

1.1 Introduction

The electric power industry in many parts of U.S.A. is undergoing a radical change from a regulated monopoly toward market competition. When a utility is a regulated monopoly, it has a guaranteed fair rate of return in exchange for an obligation to serve. In market competition, however, the utility may not expect any guarantee of fair rate of return.

In such a case, the increased degree of uncertainty has a substantial impact on generation planning aspect of the electric utility. For example, if market conditions become unfavorable (e.g., a lower electric power price), then a generation unit that has been profitable may no longer remain so. This uncertainty implies substantial financial risks in generation planning.

We note that the generation aspect is an important operation for numerous utilities. In 1997, the amount of power generated in U.S.A. was 3123 billion kWh's, resulting in \$215 billion worth of retail sales (see e.g., U.S. Bureau of the Census, 1999). Therefore, under the new economic uncertainties such as price uncertainties, it is highly desirable to help utilities in their decision making process for generation planning.

Under the new economic uncertainties, the two factors in decision making that have become much more significant are: financial risks and managerial flexibility (e.g., the values of strategic options). For example, under uncertain, competitive market conditions, expected values such as expected profit and expected rate of return have become much less meaningful without the corresponding variances (i.e., financial risks). In addition, under uncertainty, the realization of cash flows may differ significantly from what a utility may have expected

initially. As new information arrives and uncertainties about market conditions are resolved, the utility may adjust its strategy to capture future opportunities. Therefore, the managerial flexibility is also an important factor in decision making.

The traditional generation planning models, however, may not be directly applicable under the new economic uncertainties because they often have been developed under the regulated monopoly assumption and financial risks and managerial flexibility have just not been the most significant factors.

Furthermore, the traditional net present value (NPV) approach selects the projects that produce positive NPV. However, the traditional NPV is inadequate to properly capture a utility's flexibility to adapt and revise later decisions in response to unexpected market development. For example, the traditional NPV approach assumes that a new power plant will be operated continuously at a given initial scale until the end of its expected useful life.

In order to quantitatively and objectively address the two significant factors (i.e., financial risks and managerial flexibility) in generation planning, we utilize the real options approach. The real options approach can capture both financial risks and managerial flexibility in a mathematical model.

In this dissertation, we design and analyze a series of mathematical models based on the real options approach for generation planning. Hence, this dissertation can be viewed as a comprehensive study of the real options approach in generation planning. The comprehensive dissertation begins with elementary models and analyses based on the real options theories to generation planning. This is followed by more sophisticated models and analyses developed by us. Finally, to enhance relevance, various economic and cost data as well as illustrative numerical examples are presented.

The major contributions of this dissertation include the following. First, we develop comprehensive real options models for generation planning. By comprehensiveness, we mean that, with respect to the number of projects and options, we develop models that can be applied to different generation planning problems such as multiple-project single-option and multiple-project multiple-option problems. Second, we extensively study the strategic decision making based on the real options approach when a utility has multiple correlated projects. By correlation, we mean that the market values of the completed power plants are correlated. Third, we construct a lattice model to approximate multiple correlated geometric Brownian motion processes. Relative to the existing literature, our approximation model is more accurate as the true geometric Brownian motion process gets more non-linear. Fourth, we investigate the roles of correlation coefficients in decision making and the value of an option, which provides interesting managerial insights (e.g., the increase in the value of a correlation coefficient will decrease the value of an option). Finally, we compare and contrast the generation planning processes based on the real options approach and the traditional approach with respect to modeling, competition, and usage aspects.

With this dissertation, we hope that utilities may effectively and efficiently utilize the real options approach in generation planning under market uncertainties. As this approach can address the financial risks and managerial flexibility while the classical discounted cash flow approaches can not, we also hope that generation planning can be performed more quantitatively and objectively under the new economic uncertainties.

The rest of Chapter 1 is organized as follows. In Section 1.2, we show how the entire dissertation is organized. In Section 1.3, the summary of contents in each chapter of the dissertation is presented. An introduction to the traditional generation planning models is

provided in Section 1.4. Meanwhile an introduction to the theories of real options is presented in Section 1.5. Finally, in Section 1.6, we review the existing literature in traditional generation planning models and real options models.

1.2 Organization of the Dissertation

This dissertation consists of 8 chapters. Chapters 2, 3, 4, and 5 provide models based on the real options approach. Chapter 2 develops a multiple-project single-option model. Chapter 3 describes a single-project multiple-option model where the optimal options for a project are determined by backward dynamic programming.

Chapter 4 presents a two-project multiple-option model where the two projects are correlated, and investigates the role of the correlation coefficient in both decision making and value of an option. Chapter 5 extends the two-project multiple-option model to a multiple-project multiple-option model where the multiple projects are correlated. In Chapter 6, a revised traditional generation planning model that incorporates customer outage costs and utility outage costs is provided for comparison purposes to the real options models. Chapter 7 compares the real options approach to the traditional approach for generation planning. Finally, Chapter 8 summarizes this dissertation and provides directions for future research.

1.3 Summary of Contents

In Chapter 2, we develop a multiple-project single-option model. In this model, each project has an option to expand its capacity. The value of a project is represented by the expanded NPV which consists of traditional NPV and the value of the option. Then, we

select the optimal set of projects by maximizing the expanded NPV of selected projects subject to budget constraints and option availability constraints.

The multiple-project single-option model would be useful when there is essentially one dominant option for each project. However, when there are multiple options for a project, this model may not be applicable. For such cases, in Chapter 3, we develop a single-project multiple-option model. In this model, a generation planning project represents a sequence of options. The selection of an option in one period will affect the selections of options in subsequent periods. Therefore, we employ backward dynamic programming over a binomial lattice to determine the optimal options for a project and the corresponding project value.

In the single-project multiple-option model, a utility determines which option to be exercised based on market condition. Market condition is represented by the market values of a completed power plant, which can be modeled as a geometric Brownian motion (GBM) process. To avoid complex partial differential equations that may not have analytic solutions, we apply the binomial lattice model to approximate the GBM process.

The single-project multiple-option model is useful if there is essentially one important project to be considered. However, when there are multiple projects to be considered, this model may not be applicable. As a first attempt to model multiple-project problems, in Chapter 4, we develop a two-project multiple-option model where the market values of the two completed power plants are correlated. By employing backward dynamic programming over a four-branch lattice that approximates the combination of two correlated GBM processes, we determine the optimal options for the two projects and the corresponding value

of the investment. Furthermore, we investigate the roles of the correlation coefficient in the value of an option and decision making.

Next, in Chapter 5, we extend the two-project multiple-option model to a multiple-project multiple-option model. First, we approximate the combination of the multiple correlated GBM processes by a multiple-branch lattice. Then, as in Chapter 4, we employ backward dynamic programming over the lattice to determine the optimal options for the multiple projects and the corresponding value of the investment. Also, we investigate the roles of the correlation coefficients among projects in the value of an option and decision making.

Thus far, we have discussed the real options approach for generation planning. We now proceed to discuss the traditional approach for generation planning. In Chapter 6, we provide a traditional generation planning model for comparison purposes to the real options models. In this traditional model, we incorporate both forced utility outage cost and forced customer outage cost. By including the forced customer outage cost, we attempt to take the customer satisfaction level into account. Various comparisons between the real options approach and the traditional approach for generation planning are made in Chapter 7.

1.4 Introduction to Traditional Generation Planning Models

In general, the purpose of the traditional generation planning models is to determine the generation units to be constructed and the amount of power to be produced while the total cost including fixed and production cost to a utility is minimized. The cost-minimization objective functions in those models are subject to physical constraints such as demand, capacity, and reserve margin constraints. This setting may be slightly changed according to

the involvement of other operations. For example, if demand side operations such as scheduled outage during peak hours are involved, then the utility also wants to determine the amount of scheduled outage.

However, as the electric power industry is moving toward an uncertain, competitive market, the traditional models should be revised. One way to revise the traditional models is to include the outage costs of customers. In a competitive environment, customer satisfaction with the power supply may greatly influence a utility's competitive position. Customer satisfaction level can be represented by the outage costs of customers. Therefore, the inclusion of the outage costs of customers in a generation planning model will indirectly reflect the current competitive environment.

1.5 Introduction to the Theories of Real Options

Under competition, in addition to the consideration of risks, the incorporation of a utility's flexibility in generation planning is also highly desirable. The options approach can quantify the values of flexibility to utilities and support the utilities' project planning and management decisions under uncertainties more objectively.

Basically, the theory of options is a subfield of finance. Options are classified into two types: calls and puts. Call options give the option holder the right to buy a fixed number of shares of a traded stock for a fixed price on or before a given date. Put options give the holder the right to sell the stock for a fixed price on or before a given date. If the option may be exercised before maturity, it is called an American option. If the option can be exercised only at maturity, it is called a European option.

An option is a derivative asset of an underlying asset. Therefore, the value of an option depends on the value of the underlying asset. For example, the underlying asset of a stock call/put option is the stock. If the value of the stock increases, then the value of the call increases and the value of the put decreases.

There exists an analogy between financial options and opportunities for corporate investments. The opportunities for corporate investments can be viewed as financial opportunities because the corporates have the right, but not the obligation, to acquire the assets of a business. Such opportunities for corporate investments are referred to as real options.

Based on the real options theory, we can quantitatively determine the value of an option. This will provide a more accurate estimate of the value of a project when the project has operation flexibility. We believe that this enhanced understanding of the values of projects will provide better decision support in project selection and management under uncertainties. For example, for a project with multiple options at each decision time point, we can determine the optimal decisions under different market conditions by maximizing the value of a project over the planning horizon (i.e., sequential decision making under uncertainties).

1.6 Literature Review

In this section, we review the literature in traditional generation planning models and real options models that are relevant to this dissertation.

1.6.1 Traditional Generation Planning Models

Anderson [4] reviews different types of mathematical programming models that have been used for generation planning. Ammons and McGinnis [2] develop a comprehensive generation expansion planning model to determine the generation units to be constructed and the production level of each new and existing generation unit. Belgari and Laughton [8] and Sawey and Zinn [45] develop models for large-scale generation planning for the combination of generation and transmission operations. We note that these papers only focus on relatively easy-to-quantify factors such as fixed and production cost.

A few studies in the literature have considered hard-to-quantify factors such as customer outage cost and utility outage cost. Hobbs [20] incorporates scheduled utility outage cost and scheduled customer outage cost in the objective function of a model (i.e., the amount of the scheduled outage is a decision variable). Wang and Min [54], however, consider forced utility outage cost (cf. scheduled outage cost of [20]) as a component of the total cost to a utility. We note that unlike scheduled outage cost, the forced utility outage cost may be substantial.

In Chapter 6 of this dissertation, we extend Wang and Min's generation planning model by considering the forced customer outage cost in addition to the forced utility outage cost. By including the forced customer outage cost, we take the customer satisfaction level into account. Therefore, the competition in the electric power industry can be indirectly reflected in the setting of a traditional generation planning model.

1.6.2 Real Options Models

Black and Scholes [10] develop a mathematical model called Black-Scholes option-pricing formula that prices the value of an option in traditional areas of finance. This is

considered as a major development in options theories. Luehrman [33], on the other hand, shows an analogy between financial options and opportunities for corporate investments. Such opportunities for corporate investments are referred to as real options. Trigeorgis [51] summarizes common real options as the option to defer, time-to-build option, option to alter operating scale (e.g., to expand, to contract, to shut down and restart), option to abandon, option to switch (e.g., outputs or inputs), and growth options.

Luehrman [33] considers a project to build a new large-scale chemical plant and the project has an option to expand the plant's capacity three years later. The value of the option to expand can be obtained by using the Black-Scholes option-pricing table, which is developed based on the Black-Scholes option-pricing formula. Then, the value of the project is equal to the sum of the traditional NPV of the project and the value of the option. This framework is applied to develop our multiple-project single-option model in Chapter 2. Via our model, we select the optimal set of projects by maximizing the total value of the selected projects subject to constraints such as budget requirements.

We note that Black-Scholes formula is insufficient in the cases of more sophisticated models (e.g., multiple-project, multiple-option) because the underlying differential equations become quite difficult to solve. A popular way to handle these sophisticated models is by employing binomial lattice models (see e.g., [3]). Several papers apply the binomial lattice model to determine the optimal sequential options for a project. Pickles and Smith [41] apply the binomial lattice model to petroleum properties. Kelly [27] applies the same model to mining properties. Teisberg [49] analyzes the construction stage of a generation planning project using binomial lattice model. In her model, the plan to construct a power plant provides options to proceed, to delay, and to abandon. In our single-project multiple-option

model in Chapter 3, we extend the Teisberg's model to include both construction stage and operating stage of a generation planning project. By using the binomial lattice model, we determine the optimal options for the project under different market conditions and the corresponding project value.

In contrast to the single-project multiple-option models considered in Teisberg [49] or in Chapter 3, a utility may consider multiple-project models. Namely, the utility may evaluate multiple correlated generation planning projects simultaneously. For example, a utility may consider a project for a potential new gas turbine power plant and a project for a potential new wind power plant. However, few papers in the literature have addressed the issue of correlated projects based on real options. The only paper of which we are aware is for mutually exclusive R&D projects [15], which may not be an appropriate relationship for generation planning projects. Based on this part of the literature review, in Chapter 5, we have developed a four-branch lattice model for two correlated projects. In Chapter 6, we have further extended the two-correlated-project model to a multiple-correlated-project model.

CHAPTER 2. GENERATION PLANNING FOR MULTIPLE-PROJECT SINGLE-OPTION CASES

2.1 Introduction

In this chapter, we consider the following problem. Suppose there are multiple kinds of generation units. Let a project be constructing a particular generation unit. Hence, we can state that there are a multiple number of projects. Once a generation unit is constructed, then we assume that there is an option to expand (for each generation unit constructed). Hence, we can state that each project has a single option. We will refer to this problem as a multiple-project single-option case.

Under this circumstance, the value of a project consists of the traditional NPV plus the value of the option to expand. Hence, the value of a project is represented by an expanded NPV. By maximizing the expanded NPV subject to budget constraints and option availability constraints, we show how the optimal set of generation planning projects is selected.

The rest of this chapter is organized as follows. In Section 2.2, the Black-Scholes option-pricing model, which forms the basis for the multiple-project single-option model, is presented. Moreover, in Section 2.3, we present the Black-Scholes option-pricing table based on the aforementioned model, which is widely used in practice for simplicity. In Section 2.4, we formulate an expanded NPV model as a mixed-integer linear programming problem. In Section 2.5, we provide a numerical example illustrating the key features of our model. Finally, concluding remarks are provided in Section 2.6.

2.2 Black-Scholes Option-Pricing Model

As mentioned previously, the traditional NPV approach cannot properly reflect utility's flexibility to adapt and revise later decisions in response to unexpected market developments. However, this does not mean that NPV should be discarded because it does contain relevant revenue and cost information. Instead, NPV should be regarded as a required input to an option-based, expanded NPV analysis. Suppose that a generation expansion planning project (to construct a generation unit) has an option to expand its capacity. Then, by the definition of expanded NPV in Luehrman [33], the value of the entire project is given by:

Expanded NPV (entire project) = traditional NPV (Phase I assets) + present value of an option (Phase II assets)

Phase I assets refer to the initial investment for a new generation unit and subsequent net cash flows. Phase I assets can be valued using the traditional NPV approach. Phase II assets refer to the value of an option to expand created by the initial investment. In this section, we will focus on how to estimate the value of an option employing the Black-Scholes option-pricing model (for underlying technical assumptions and the derivation of the model, see Trigeorgis [51], Luenberger [34], Hull [23]).

The Black-Scholes option-pricing model provides the option's value. When the five inputs--present value of the underlying asset (the assets to be acquired when and if the utility exercises the option), value of construction expense required for exercising the option, risk-free rate of return, time to the decision date, and volatility of the underlying asset per unit of time--are specified, then the present value of the option to expand is given by equation (2.1). We note that, to obtain equation (2.1), it is assumed that the value of the underlying asset

follows geometric Brownian motion and the expected value of the option is discounted with respect to risk-neutral probability (the details will be presented in Chapter 3).

$$V = N(d_1)S - N(d_2)Xe^{-rt} \quad (2.1)$$

where

V = present value of the option to expand (i.e., Phase II assets)

$N(d_1)$ and $N(d_2)$ are the values of the cumulative standard normal distribution

at d_1 and d_2 , where

$$d_1 = [\ln(\frac{S}{X}) + (r + 0.5\sigma^2)t] / \sigma\sqrt{t}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

S = present value of the underlying assets (can be observed in the market)

X = value of construction expense required for exercising the option (defined by the features of the investment)

r = risk-free rate of return (can be observed in the market)

t = time to the decision date (defined by the features of the investment)

σ = volatility of the underlying asset per unit of time (it is often the only estimated input)

$N(d_1)S$ can be interpreted as the expected value of S if $S > X$ on the decision date.

Xe^{-rt} can be interpreted as the present value of X . $N(d_2)Xe^{-rt}$ can be interpreted as the expected value of Xe^{-rt} if $S > X$ on the decision date. Hence, the present value of the option to expand is given by the expected value of the present value of the underlying asset minus

the expected value of the present value of construction expense required for exercising the option.

From the above equation, other factors being constant, the value of the option to expand is higher if the (1) present value of the underlying asset, S , is higher; (2) time to the decision date, t , is longer; (3) value of construction expense required for exercising the option, X , is lower; (4) variance of the underlying asset, σ^2 , is higher; and (5) risk-free rate of return, r , is higher.

2.3 Black-Scholes Option-Pricing Table

Because of the computational complexity, the Black-Scholes option-pricing model is converted to a table (called the Black-Scholes option-pricing table). In this section, we will employ the Black-Scholes option-pricing table for the value of a European call option (we are viewing an option to expand as a European call option because it can be exercised only at maturity). Part of the Black-Scholes option-pricing table for the value of a European call option is shown in Table 2.1 (i.e., the complete table can be found in [51]).

Table 2.1 Black-Scholes option-pricing table for the value of a European call option

		NPVq														
		0.80	0.82	0.84	0.86	0.88	0.90	0.92	0.94	0.96	0.98	1.00	1.02	1.04	1.06	1.08
$\sigma\sqrt{t}$	0.50	11.8	12.6	13.4	14.2	14.9	15.7	16.5	17.3	18.1	18.9	19.7	20.5	21.3	22.1	22.9
	0.55	13.8	14.6	15.4	16.1	16.9	17.7	18.5	19.3	20.1	20.9	21.7	22.4	23.2	24.0	24.8
	0.60	15.8	16.6	17.4	18.1	18.9	19.7	20.5	21.3	22.0	22.8	23.6	24.3	25.1	25.8	26.6
	0.65	17.8	18.6	19.3	20.1	20.9	21.7	22.5	23.2	24.0	24.7	25.3	26.2	27.0	27.7	28.4
	0.70	19.8	20.6	21.3	22.1	22.9	23.6	24.4	25.2	25.9	26.6	27.4	28.1	28.8	29.5	30.2

The same five inputs as for the Black-Scholes option-pricing model are needed for the value of an option to expand via the Black-Scholes option-pricing table. This table uses $NPVq$ and $\sigma\sqrt{t}$ to find the value of an option to expand as a percentage of the present value of the underlying asset. The subscript q of $NPVq$ represents a quotient. $NPVq$ is expressed as $S/PV(X)$. We note that $PV(X)$ represents the present value of X . Since σ^2 denotes the variance of the underlying asset per unit of time, the multiplication of σ^2 and t provides the cumulative (over time) variance of the underlying asset. The square root of the cumulative variance, $\sigma\sqrt{t}$, is called the cumulative volatility of the underlying asset.

In Table 2.1, each number represents the value of an option to expand as a percentage of the present value of the underlying asset. Luehrman [33] says, “Option values in this table are expressed in relative terms, as percentages of S , rather than in absolute dollars, to enable us to use the same table for both big and small projects.”

To demonstrate how to use the Black-Scholes option pricing table, we provide a numerical example for the value of an option to expand. The five inputs for this example are $S = \$12$ million, $X = \$16$ million, $r = 4\%$, $t = 4$ years, and $\sigma = 30\%$. The present value of the exercise price can be calculated as $PV(X) = 16 / (1 + 0.04)^4 = 13.6769$. Therefore, we have $\sigma\sqrt{t} = 0.6$ and $NPVq = 12 / 13.6769 = 0.877$. From Table 2.1, the value of the option is 18.8% (by interpolation) of the present value of the underlying asset. Thus, the value of the option to expand is equal to \$2.256 million ($0.188 * 12$).

To demonstrate the accuracy of the Black-Scholes option pricing table, we compare the above result with the result obtained from the Black-Scholes option pricing formula. To

calculate the value of the option using the Black-Scholes option pricing formula, we first calculate d_1 and d_2 as

$$d_1 = [\ln(12/16) + (0.04 + 0.5 * 0.3^2) * 4] / 0.3 * \sqrt{4} = 0.0872$$

$$d_2 = d_1 - 0.3 * \sqrt{4} = -0.513$$

Then, we use the standard normal distribution table to obtain

$$N(0.0872) = 0.5347, N(-0.513) = 0.304$$

Finally, we calculate the value of the option to expand as

$$0.5347 * 12 - 0.304 * 16 * e^{-0.04 * 4} = \$2.272 \text{ million}$$

By comparing 2.256 with 2.272, we recognize that the error of using the Black-Scholes option pricing table is less than one percent (i.e., we consider the value via the Black-Scholes option pricing formula as the true value). Therefore, we conclude that the Black-Scholes option pricing table produces a reasonable result with relatively less computational efforts.

2.4 An Expanded NPV Maximization Model

Thus far, we have shown how to obtain the value of an option via the Black-Scholes option-pricing table. In this section, we will formulate an expanded NPV maximization model for generation expansion planning. We assume that there are multiple projects where each project has one option to expand (i.e., a European call option).

In this model, we maximize the sum of Phase I assets (traditional NPV) and Phase II assets (the present value of an option) subject to budget constraints and option availability constraints. We note that option availability indicates that it is feasible to exercise an option

or to choose not to exercise an option. The corresponding decision variables are binary variables for generation expansion planning projects, binary variables for options, and the amount of construction budget not used in each period. Therefore, we have a mixed-integer linear programming model as follows

Maximize

$$E_t, y_j, z_j$$

$$\sum_{j=1}^J (NPV_{ij} y_j + P_{ij} S_{ij} z_j)$$

Subject to

$$E_t - (1+r)E_{t-1} + \sum_{j=1}^J X_{ijt} y_j + \sum_{j=1}^J X_{ijt} z_j = B_t \quad (2.2)$$

$$z_j \leq y_j \quad (2.3)$$

where

j = index of generation expansion planning project, $j = 1, \dots, J$

NPV_{ij} = traditional NPV for project j (Phase I assets) (\$)

= $S_{ij} - X_{ij}$, where

S_{ij} = present value of project j 's Phase I assets (assets to be acquired for project j) (\$)

X_{ij} = present value of the construction expense required to construct project j 's Phase I assets (\$)

$$= \sum_{t=1}^T \frac{1}{(1+r)^t} X_{ijt}$$

y_j = binary variable, indicating if generation expansion planning project j is

to be constructed ($y_j=1$ implies construction, $y_j=0$ implies no construction)

P_{ij} = value of the option to expand as a percentage of the present value of the assets to be acquired when and if the utility exercises the option for project j (i.e., P_{ij} can be found in Table 2.1)

S_{ij} = present value of the assets to be acquired when and if the utility exercises the option for project j (i.e., the present value of the underlying asset) (\$)

$P_{ij}S_{ij}$ = present value of the option for project j (Phase II assets) (\$)

z_j = binary variable, indicating if the option for project j is available ($z_j = 1$ implies available, $z_j = 0$ implies not available)

E_t = construction budget unused in period t (\$), $E_0 = 0$

r = discount rate

X_{ijt} = construction expense required to construct project j 's Phase I assets in period t (\$)

X_{ij} = present value of the construction expense required to exercise the option to expand for project j (\$)

$$= \sum_{t=1}^T \frac{1}{(1+r)^t} X_{ij,t}$$

$X_{ij,t}$ = construction expense required to exercise the option to expand for project j in period t (\$)

B_t = construction budget available in period t (\$)

t = index of period of time, e.g., year, $t = 1, \dots, T$

The objective function is to maximize the sum of Phase I assets and Phase II assets of selected projects. It is measured in present worth because the planning horizon can be long (ten years or more).

The budget constraints (2.2) state that in each period a certain amount of budget is available for the construction expense required for generation expansion planning projects and for exercising the options. The budget that is left in the current period will be carried forward to the next period for construction. The budget carried forward earns interest at some specified rate of return on short-term investment. We note that E_t , B_t , X_{ijt} , and X_{ijt} in these constraints are not discounted to the present value.

The option availability constraints (2.3) state that the option for project j is available only if project j is selected. We note that option availability indicates that it is feasible to exercise an option or to choose not to exercise an option. If project j is not selected ($y_j = 0$), then the option for project j is not available ($z_j = 0$).

The capital budgeting process shown in this model has the following steps:

- (1) Select generation expansion planning projects.
- (2) After the selection of projects, construct the selected projects, given the expenses and time durations subject to budget constraints.
- (3) After the construction of selected projects, the option to expand may be available, subject to budget constraints.

The available, economically feasible expansion options will be exercised in the future after uncertainties about market conditions have been resolved (conditions under which an option will or will not be exercised can be derived with more sophisticated models; see Trigeorgis [51]).

2.5 An Example

We now provide a numerical example with hypothetical data to illustrate the applicability of our model. In this example, there are three generation expansion planning projects. Each project has one option to expand its capacity. The planning horizon is 7 years. Project 1 is to build generation unit 1 immediately, and the construction of generation unit 1 will create an option to expand 3 years later. Project 2 is to build generation unit 2 immediately, and the construction of generation unit 2 will create an option to expand 4 years later. Project 3 is to build generation unit 3 immediately, and the construction of generation unit 3 will create an option to expand 5 years later. We note that the entire planning horizon is 7 years in this example. However, this planning horizon can be easily extended.

The rate of return is assumed to be 10%. The budgets for years 1, 2, 3, 4, 5, 6, and 7 are \$250, 100, 100, 400, 50, 50, and 0 million, respectively. The volatility of the assets (for expansion of capacity) to be acquired when and if the utility exercises the option for Projects 1, 2, and 3 is assumed to be 0.4, 0.3, and 0.25 (in \$ millions), respectively. The net cash flows for Projects 1, 2, and 3 are listed in Tables 2.2, 2.3, and 2.4, respectively. We note that the figures in these tables are in millions of dollars. We also note that the cash flows in the rows of Phase I and Phase II in Tables 2.2, 2.3, and 2.4 are not discounted.

Table 2.2 Net cash flows for Project 1

Year	1	2	3	4	5	6	7
Phase I							
Present Value	-125	8	8	7.8	7.3	6.9	103.2
Phase II							
Present Value				-433			
Present Value				-325.3	14.7	18.4	226.6

Table 2.3 Net cash flows for Project 2

Year	1	2	3	4	5	6	7
Phase I							
Present Value	-60	-50	-30				
Present Value	-60	-45.5	-24.8	15	15	30	40
Phase II							
Present Value					-58.6		
Present Value					-40	20	20

Table 2.4 Net cash flows for Project 3

Year	1	2	3	4	5	6	7
Phase I							
Present Value	-50	-50					
Present Value	-50	-45.5	20	20	20	20	40
Phase II							
Present Value						-193.3	
Present Value						-120	100

First, we calculate Phase I assets for each project. Second, we calculate the parameters $NPVq$ and $\sigma\sqrt{t}$ required for the Black-Scholes option-pricing table. Third, by using $NPVq$ and $\sigma\sqrt{t}$ as well as the Black-Scholes option-pricing table, we obtain a percentage (of the present value of the underlying asset) for the option to expand for each project. We note that if the Black-Scholes option-pricing table does not show values that

correspond exactly to our calculated values for NPV_q and $\sigma\sqrt{t}$, then we may employ interpolation to obtain a percentage. Next, the Phase II assets for each project can be obtained by multiplying this percentage by the present value of the assets to be acquired when and if the utility exercises the option (S_{IIj}). Finally, the sum of Phase I assets and Phase II assets provides the expanded NPV. The results of these calculations are listed in Table 2.5.

Table 2.5 Expanded NPV for each project

Project 1	Project 2	Project 3
$S_{I1} = 141.2$	$S_{I2} = 100$	$S_{I3} = 120$
$X_{I1} = 125$	$X_{I2} = 130.3$	$X_{I3} = 95.5$
$NPV_{I1} = 16.2$ (Phase I)	$NPV_{I2} = -30.3$ (Phase I)	$NPV_{I3} = 24.5$ (Phase I)
$S_{II1} = 259.7$	$S_{II2} = 40$	$S_{II3} = 100$
$X_{II1} = 325.3$	$X_{II2} = 40$	$X_{II3} = 120$
$NPV_q = \frac{259.7}{325.3} = 0.80$	$NPV_q = \frac{40}{40} = 1$	$NPV_q = \frac{100}{120} = 0.83$
$\sigma\sqrt{t} = 0.4\sqrt{3} = 0.693$	$\sigma\sqrt{t} = 0.3\sqrt{4} = 0.6$	$\sigma\sqrt{t} = 0.25\sqrt{5} = 0.56$
$P_{II1} = 0.195$	$P_{II2} = 0.236$	$P_{II3} = 0.154$
$P_{II1}S_{II1} = 50.64$ (Phase II)	$P_{II2}S_{II2} = 9.44$ (Phase II)	$P_{II3}S_{II3} = 15.4$ (Phase II)
Expanded NPV = 66.84	Expanded NPV = -20.86	Expanded NPV = 39.9

We now input the results in Table 2.5 into our model. A mixed-integer linear programming problem is shown as follows:

Maximize

E 's, y 's, z 's

$$16.2y_1 - 30.3y_2 + 24.5y_3 + 50.64z_1 + 9.44z_2 + 15.4z_3$$

Subject to

$$E_1 + 125y_1 + 60y_2 + 50y_3 = 250$$

$$E_2 - 1.1E_1 + 50y_2 + 50y_3 = 100$$

$$E_3 - 1.1E_2 + 30y_2 = 100$$

$$E_4 - 1.1E_3 + 433z_1 = 400$$

$$E_5 - 1.1E_4 + 58.6z_2 = 50$$

$$E_6 - 1.1E_5 + 193.3z_3 = 50$$

$$E_7 - 1.1E_6 = 0$$

$$z_1 \leq y_1$$

$$z_2 \leq y_2$$

$$z_3 \leq y_3$$

$$y_1, y_2, y_3, z_1, z_2, z_3 \in \{0,1\}$$

Employing the LINDO software package [29], the optimal solution is $y_1 = 1, y_2 = 0, y_3 = 1, z_1 = 1, z_2 = 0, z_3 = 1, E_1 = 75, E_2 = 132.5, E_3 = 245.75, E_4 = 237.32, E_5 = 311.06, E_6 = 198.86, E_7 = 218.75$. Hence, Projects 1 and 3 are selected, and the options for Projects 1 and 3 are available. The corresponding objective function value is \$106.74 million, which is composed of the expanded NPV of \$66.84 million for Project 1 and the expanded NPV of \$39.9 million for Project 3.

We note that, by the traditional NPV approach, the NPV of Projects 1, 2, and 3 is -\$49.4 million, -\$30.3 million, and \$4.5 million, respectively (under traditional NPV, the expansion is not an option). Hence, in such a case, only Project 3 may be selected. By the real options approach, the expanded NPV of Projects 1, 2, and 3 is \$66.84 million, -\$20.86

million, and \$39.9 million, respectively. Hence, Projects 1 and 3 are attractive. The main cause for this difference (selecting Project 3 only vs. selecting Projects 1 and 3) is that, under the real options approach, uncertainty creates opportunities, and opportunities add values to a project (the value of an option is nonnegative).

The Black-Scholes option-pricing model and the subsequent Black-Scholes option-pricing table are well suited for simple real options with a single decision date. However, for those complex real options with many decision dates, special mathematical tools such as the binomial option valuation model are required (see Amram and Kulatilaka [3]).

2.6 Concluding Remarks

In this chapter, we formulated and analyzed a generation expansion planning model for electric utilities based on the options theory. A critical contribution of our model was the incorporation of the traditional NPV and the present value of an option to expand in the objective function of the planning model. By maximizing the expanded NPV subject to budget constraints and option availability constraints, we showed how the optimal set of generation units to be constructed is determined.

In developing this model, we first started with a description of the Black-Scholes option-pricing model for the value of an option to expand. Next, we presented the Black-Scholes option-pricing table based on the Black-Scholes model, which is widely used in practice. This table is used to compute the value of the option that is a component in the objective function. Once the generation expansion planning model was constructed based on the options theory, we provided a numerical example illustrating the key features of our model.

In this model, we considered budget and option availability constraints. Other constraints that can be included in the future are: capacity constraints, reserve margins, and reliability requirements.

CHAPTER 3. A SINGLE-PROJECT MULTIPLE-OPTION MODEL FOR GENERATION PLANNING

3.1 Introduction

In Chapter 2, we studied the multiple-project single-option model. In contrast, in this chapter, we investigate the case of single-project multiple-option.

A single project here represents a potential new generation unit. The multiple options are, for example, to construct a new generation unit, to defer construction, or to expand the constructed capacity level (see e.g., [56]). We note that options may change according to the past history of action (or inaction), time, and market conditions.

Given our definitions of a project and options, we will help utilities to determine the optimal sequential decisions over options (to be exercised) for a project. The options mentioned here are of strategic nature, and the market conditions here are represented by the market value of a completed generation unit. This market value will be utilized as the underlying asset for the strategic options.

In this chapter, the movement of the market value of the completed generation unit is assumed to follow the geometric Brownian motion process (see e.g., [48] [49]). Since an option is a derivative asset of an underlying asset, the optimal sequential decisions over options for the project are determined according to the movement of the market value of the completed generation unit.

In the multiple-project single-option model in Chapter 2, we show how to obtain the value of an option by using the Black-Scholes option-pricing model (table). However, since our problem involves many options with respect to time, the partial differential equations

derived from geometric Brownian motion process are too complex to have analytic solutions. Therefore, we employ a binomial lattice model to approximate the geometric Brownian motion process (see e.g., [23], [34]). In other words, we assume that the uncertain market conditions can be now represented by a binomial lattice model.

Since we will be sequentially making decisions over options, we will utilize a backward dynamic programming formulation (see e.g., [26]). By maximizing the overall project value during the planning horizon, we will determine the optimal sequential decisions over the options.

This chapter is organized as follows. In Section 3.2, the real options for generation planning are presented. Section 3.3 presents the geometric Brownian motion process that represents the evolution of the market value of a completed generation unit. Section 3.4 shows how to approximate a geometric Brownian motion process by a binomial lattice model. Section 3.5 provides the dynamic programming formulation. An illustrative example is presented for managerial insights and economic implications in Section 3.6. Finally, concluding remarks are provided in Section 3.7.

3.2 Real Options for Generation Planning

A generation planning project here represents a sequence of options. The options can be divided into two stages. The first stage is the construction stage. During this stage, a utility has the options to construct a generation unit and to defer the construction. The second stage is the operating stage. During this stage, a utility has the options to operate at the constructed capacity level, to expand the constructed capacity level, to operate at the

expanded capacity level, and to temporarily shut down a generation unit from its constructed capacity level or expanded capacity level.

The construction of a generation unit may require several periods (e.g., years) to construct. If market conditions are favorable (e.g., the value of the completed generation unit is sufficiently large) at the beginning of a period, then a utility may start or continue the construction of a generation unit in the same period. During the period of construction, net cash flow becomes negative because construction costs occur and no revenues are obtained yet. On the other hand, if market conditions are unfavorable (e.g., the value of the completed generation unit is sufficiently small) at the beginning of a period, then a utility may defer the construction in the same period and wait for new information. During the period of deferring, net cash flow is assumed zero.

A utility starts the operation stage of a generation unit when its construction is completed. At the operation stage, the expected net cash flow in a period (i.e., the expected profit) is calculated as the difference between the revenue from electric power sales and the cost of operation in the same period. If the market condition is as expected (i.e., the value of the generation unit is not sufficiently high or low) at the beginning of a period, then the utility may operate the generation unit at its constructed capacity in the same period. If the market conditions are far more favorable than expected at the beginning of a period (i.e., the value of the generation unit is sufficiently high), then a utility may expand constructed capacity level in the same period. During this period of the expansion, a utility operates a generation unit at its constructed capacity level and constructs the expansion at the same time. The total costs are equal to the sum of production costs and expansion costs. The net

cash flow now is equal to the difference between the revenue from selling electricity and the total costs.

If market conditions are far less favorable than expected at the beginning of a period (i.e., the value of the generation unit is sufficiently low), then a utility may temporarily shut down the generation unit in the same period and wait for new information. During the period of shutdown, net cash flow is assumed zero. If new information is favorable, then the generation unit may be switched from shutdown to operation. If the generation unit had been operated at the constructed capacity level before it was shut down, then it would be switched to operation at the constructed capacity level. Similarly, if the generation unit had been operated at the expanded capacity level, we assume that it would be switched to operation at the expanded capacity level (we can also assume that it would be switched to operating at the original capacity level without much difficulty).

Switching from one option to another option requires a switching cost. For example, switching from shutdown to operating entails a setup cost. Some switches are not feasible. For example, switching from operating at a constructed capacity level to deferring construction is not feasible. Therefore, the switching cost in this case is assumed to be infinite.

3.3 Geometric Brownian Motion

Market conditions can be represented by the market value of a completed generation unit. For example, if the market value of a completed generation unit decreases dramatically in a period, then deferring may be a good option in the next period. However, at time 0 when strategic planning is made for a generation planning project, the generation unit that the

project represents does not exist. To estimate the market value of a completed generation unit at time 0, we may use a currently existing generation unit that has similar features such as capacity and construction costs (see e.g., [40], [48]).

The market value of the completed generation unit is considered as the major source of uncertainties after time 0. Let X be the market value of the completed generation unit. According to Teisberg [48], [49], it is reasonable to assume that X evolves stochastically over time according to a geometric Brownian motion (GBM) shown in the following equation.

$$dX = \mu X dt + \sigma X dz$$

where dz is the differential of a standard Wiener process (with mean 0 and variance dt) for the completed generation unit. μ is the instantaneous expected rate of return on the completed generation unit. σ is the instantaneous standard deviation of the rate of return on the completed generation unit.

3.4 Binomial Lattice Model

Instead of developing complicated differential equations that may not have analytic solutions, we approximate a continuous GBM by a discrete binomial lattice model. In a binomial lattice model, the change in the value of X has two possibilities: going up or going down.

These two possible values are defined as the multiples of the value of X in the previous period as shown in Figure 3.1 – a multiple u for up ($u > 1$) and a multiple d for down ($d < 1$), and $ud = 1$. The probabilities for up and down are p and $1 - p$, respectively.

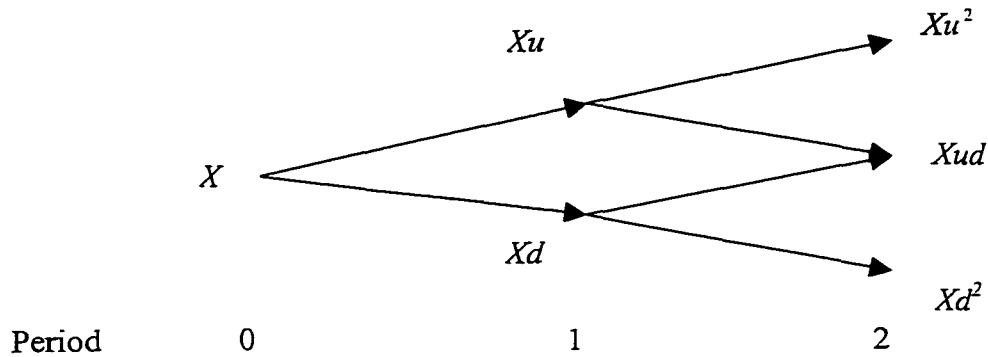


Figure 3.1 Binomial lattice for underlying asset

We note that the index of periods in Figure 3.1 represents the end of that period or the beginning of the next period.

Approximating a GBM by a binomial lattice model requires two steps. The first step is to transform the GBM for X to a process for $\ln X$. The second step is to find the suitable values for u, d , and p_u by matching the binomial lattice model to the process for $\ln X$.

A. Transformation

Let $W = \ln X$, then, we keep terms in dW up to first order in dt as in equation (3.1).

$$\begin{aligned}
 dW &= \frac{dW}{dX} dX + \frac{1}{2} \frac{d^2W}{dX^2} (dX)^2 \\
 &= \frac{1}{X} (\mu X dt + \sigma X dz) \\
 &\quad - \frac{1}{2} \frac{1}{X^2} (\sigma^2 X^2 dt)
 \end{aligned} \tag{3.1}$$

It can be shown that $(dz)^2$ is equal to dt . We note that the only nonzero term in $(dX)^2$ is $\sigma^2 X^2 dt$. Therefore, we have

$$d \ln X = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dz \quad (3.2)$$

B. Matching

We match the expected value of the change in $\ln X$ in the binomial lattice model to the expected value of $d \ln X$ in equation (3.2). Also, we match the variance of the change in $\ln X$ in the binomial lattice model to the variance of $d \ln X$ in equation (3.2).

Let $X(t + \Delta t)$ be the value of X at time $t + \Delta t$. $\ln X(t + \Delta t)$ in the binomial lattice model can be expressed as

$$\ln X(t + \Delta t) = \ln X(t) + Y$$

where

$$Y = \begin{cases} \ln u & \text{with probability } p \\ \ln d & \text{with probability } 1 - p \end{cases}$$

Therefore, the matching is as follows:

$$\begin{aligned} E[Y] &= p \ln u + (1 - p) \ln d \\ &= \left(\mu - \frac{1}{2}\sigma^2\right)\Delta t \\ \text{var}(Y) &= E[Y^2] - (E[Y])^2 \\ &= (\ln u)^2 4p(1 - p) = \sigma^2 \Delta t \end{aligned}$$

The suitable values for u , d , and p are solved as in [34]

$$p = \frac{1}{2} + \frac{1}{2} \left(\frac{\mu - \frac{1}{2}\sigma^2}{\sigma} \right) \sqrt{\Delta t} \quad (3.3)$$

$$u = e^{\sigma\sqrt{\Delta t}} \quad (3.4)$$

$$d = e^{-\sigma\sqrt{\Delta t}} \quad (3.5)$$

We note that by replacing μ by r , risk-free rate of return, p becomes risk-neutral probability (for the development of risk-neutral probability, see Appendix A).

3.4.1 Net Cash Flows of the Options

Each option has a binomial lattice to represent net cash flows that an option would generate if it is exercised. In Figure 3.2, $C_t^s(i)$ represents the net cash flow that occurs at the beginning of time t (and state s) when operating in option i .

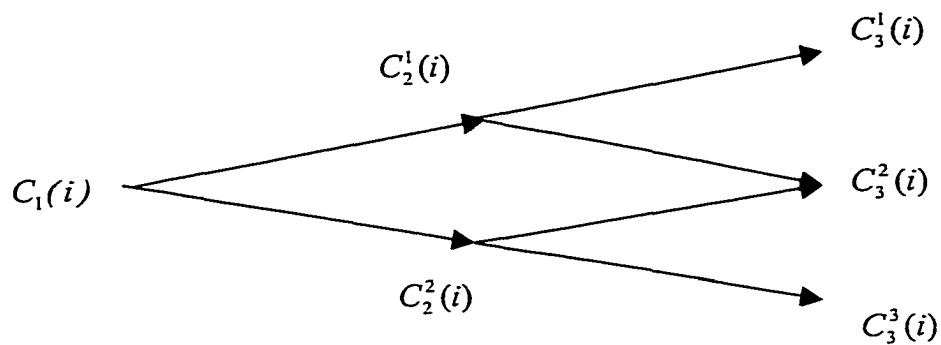


Figure 3.2 Binomial lattices for options

For some options, net cash flows are undefined in some periods. For example, if both the construction and expansion of a generation unit require one period to complete, then the earliest net cash flows for the option to operate at an expanded capacity level occur in period 2. Net cash flows are undefined (UD) in periods 0 and 1.

3.5 Other Numerical Method for the GBM

In Section 3.4, a GBM process for the underlying asset is approximated by a binomial Lattice model. However, the binomial lattice model is not the only method to approximate the GBM process. For example, Monte Carlo simulation uses random numbers to sample many different paths that the underlying asset could follow (see e.g., [23]). For each path, the net cash flow of an option can be calculated and discounted at the risk-free rate of return.

We note that simulation can provide numerical solution to the value of an option. However, as we proceed to more complicated models, such as two-project/multiple-project models in Chapters 4, 5, it would be difficult to apply numerical methods such as simulation to do further analysis (e.g., investigation of the roles of correlation coefficients among projects).

3.6 Dynamic Programming

Because the selection of an option in one period will affect the selections of options in subsequent periods, the use of backward dynamic programming is necessary. We solve the generation planning problem recursively, starting from the end and moving back, to determine the optimal operating options for a project and the corresponding project value.

We apply the recursive relationship in Trigeorgis [51]. The project value at the beginning of t , given state s is newly observed while option during $t - 1$ was m , is

$$E_t^s(m) = \max_i \left(C_t^s(i) + \frac{pE_{t+1}^s(i) + (1-p)E_{t+1}^{s+1}(i)}{1+r} - I(m \rightarrow i) \right) \quad (3.6)$$

with $I(m \rightarrow i) = 0$ for $i = m$

where

s	=	index of states
i	=	index of options
t	=	index of periods
$E_t^s(m)$	=	project value at the beginning of t , given state s is newly observed while option during $t - 1$ was m (assuming optimal future decisions)
$C_t^s(i)$	=	net cash flow at the beginning of t , given state s is newly observed while operating in option i
p	=	risk-neutral probability
r	=	risk-free rate of return
$I(m \rightarrow i)$	=	switching cost from option m to option i

Let T denote when the backward process starts. At $t = T$, the above recursive relationship becomes

$$E_T^s(m) = \max_i (C_T^s(i) - I(m \rightarrow i)) \quad (3.7)$$

3.7 Numerical Example

3.7.1 Input Data

We now provide a numerical example with hypothetical data to illustrate the applicability of our model. In this example, there is a generation planning project for a gas turbine generation unit. The project has seven options and has a 3-period planning horizon.

The length of a period, Δt , is assumed to be one year. Furthermore, we assume that each activity (e.g., construction, expansion, etc.) will last one year for simplicity.

The constructed capacity level is assumed to be 129 MW. The expanded capacity level is assumed to be 180 MW. Risk-free rate of return, r , is assumed to be 0.08. The instantaneous standard deviation of the rate of return on the completed gas turbine generation unit, σ , is assumed to be 0.6. By using equation (3.4) and equation (3.5), the multiple factors u and d for the underlying asset (the market value of the completed gas turbine generation unit) can be calculated as 1.8 and 0.6, respectively. Also, by using equation (3.3), risk-neutral probability, p , can be calculated as 0.4.

$i = 0, 1, 2, 3, 4, 5, 6$ represent the activities on which options are based to defer the construction of the generation unit, to construct the generation unit, to operate at the constructed capacity level, to expand the constructed capacity level, to operate at the expanded capacity level, to temporarily shut down the generation unit from operating at the constructed capacity level, and to temporarily shut down the generation unit from operating at the expanded capacity level, respectively. We assume that both construction and expansion require one period to complete.

Figures 3.3 represents the binomial lattices for $i = 0, 1, 2, 3, 4, 5, 6$, respectively. In these lattices, net cash flows are expressed in \$ millions. The first term in the parentheses represents the revenue generated in a period. The second term represents the costs incurred in a period. The difference of these two terms provides net cash flow in a period.

Switching costs are expressed in \$ millions. $I(0 \rightarrow 0,1) = 0$ means that the switching costs from option 0 to 0 and from option 0 to 1 are equal to 0. The rests of switching costs are $I(0 \rightarrow 2,3,4,5,6) = \infty$, $I(1 \rightarrow 0,1,4,6) = \infty$, $I(1 \rightarrow 2,3) = 0$, $I(1 \rightarrow 5) = 0.5$,

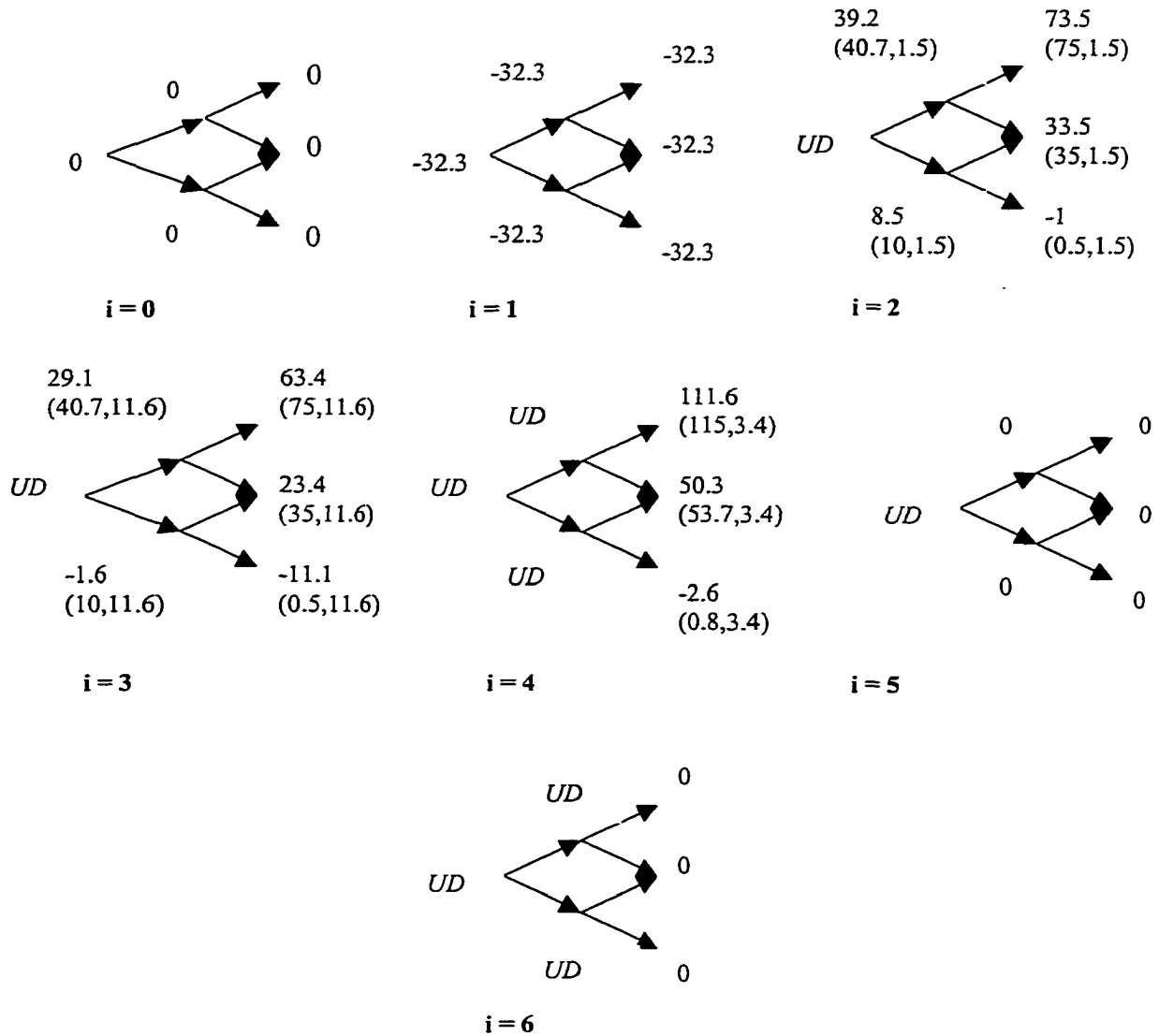


Figure 3.3 Binomial lattices for $i = 0, 1, 2, 3, 4, 5, 6$

$$\begin{aligned}
 &I(2 \rightarrow 0, 1, 4, 6) = \infty, \quad I(2 \rightarrow 2, 3) = 0, \quad I(2 \rightarrow 5) = 0.5, \quad I(3 \rightarrow 0, 1, 2, 3, 5) = \infty, \quad I(3 \rightarrow 4) = 0, \\
 &I(3 \rightarrow 6) = 1, \quad I(4 \rightarrow 0, 1, 2, 3, 5) = \infty, \quad I(4 \rightarrow 4) = 0, \quad I(4 \rightarrow 6) = 1, \quad I(5 \rightarrow 0, 1, 3, 4, 6) = \infty, \\
 &I(5 \rightarrow 2) = 0.5, \quad I(5 \rightarrow 5) = 0, \quad I(6 \rightarrow 0, 1, 2, 3, 5) = \infty, \quad I(6 \rightarrow 4) = 1, \quad I(6 \rightarrow 6) = 0.
 \end{aligned}$$

3.7.2 Solution

We solve this 3-period generation planning problem by using backward dynamic programming. We first determine the optimal operating option and the corresponding project value for each state at $t = 2$ by using equation (3.7) (i.e., the index of periods starts at $t = 0$). However, at $t = 2$, we do not have knowledge of what would be the optimal operating options at $t = 1$. Therefore, for each state at $t = 2$, we need to consider each of the seven options as a possible entering option. Each possible entering option results in one binomial lattice with a set of project values and the optimal operating options.

For each binomial lattice, at $t = 1$ and $t = 0$, we use equation (3.6) to determine the optimal operating options and the corresponding project values. We note that, at $t = 0$, option 0 is the only possible entering option. After we solve $t = 0$, we trace forward to determine the optimal operating options for this 3-period generation planning project (details of this procedure are available on request). The optimal operating options are shown in Figure 3.4. The corresponding project value at $t = 0$ is \$15.6 millions.

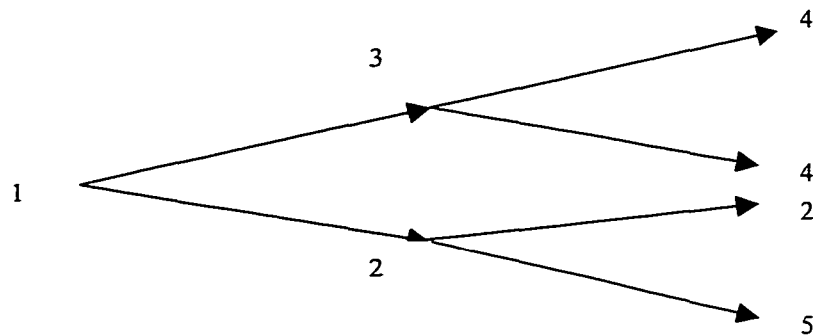


Figure 3.4 Optimal operating options for a 3-period generation planning project

We observe that , at the beginning of period 1, we should construct the 129 MW generation unit. If market conditions go up during period 1, then we should expand the constructed capacity level to 180 MW at the beginning of period 2. Whether or not market conditions go up during period 2, we should operate the generation unit at the expanded level at the beginning of period 3. On the other hand, if market conditions go down during period 1, we should operate the generation unit at the constructed capacity level at the beginning of period 2. If market conditions go up during period 2, we should still operate the generation unit at the constructed capacity level at the beginning of period 3. However, if market conditions go down during period 2, we should temporarily shut down the generation unit at the beginning of period 3.

3.8 Concluding Remarks

In this chapter, we employed a binomial options model to represent uncertain market conditions for a generation planning project. A contribution of this chapter is that we incorporated the strategic options at construction and operation stages that a utility may encounter in generation planning into a dynamic programming model. By solving the backward dynamic programming formulation, we determined the optimal sequential decisions over options for a generation planning project and the corresponding project values.

In this chapter, we focused on strategic options only. For future research, it would be interesting to consider tactical options (e.g., daily decisions on production/non-production) as well.

CHAPTER 4. A MULTIPLE-OPTION MODEL WITH TWO CORRELATED PROJECTS FOR GENERATION PLANNING

4.1 Introduction

In Chapter 3, we studied the single-project multiple-option model. When there is more than one project and each project has one dominant option, but the projects are independent, then the model in Chapter 2 can be applied. On the other hand, if the projects are correlated, then each project can not be treated separately. This chapter is motivated by the inadequacy of the previous model when the projects are correlated. By correlation here, we mean the market values of the completed generation units are correlated (see e.g., [37], [57]). As a first step toward the development of a full model, in this chapter, we will consider the case of two correlated projects.

Teisberg [49] analyzes the construction stage of a generation planning project using the real options approach. In her model, the construction for a generation unit can be proceeded, be delayed, or be abandoned. In practice, however, a utility may evaluate multiple correlated generation planning projects simultaneously. Few papers in the literature have addressed the issue of correlated projects based on real options. The only paper of which we are aware is for mutually exclusive R&D projects [15], which may not be an appropriate relationship for generation planning projects (e.g., generation projects need not be mutually exclusive).

Mathematically, the fact that the values of the completed generation units are correlated can be modeled with correlated geometric Brownian motion (GBM) processes. Furthermore, in Chapter 3, we assume that a GBM process can be approximated by a

binomial lattice model. In this chapter, we construct a four-branch lattice model to approximate the combination of two correlated GBM processes. This approximation utilizes the correlation coefficient between the increments of two Brownian motion components (see e.g., [5], [22], [47]).

Once the four-branch lattice model is constructed, we develop a mathematical expression for the value of an option. Since the expression is a function of the correlation coefficient, we then investigate the role of the correlation coefficient in the value of an option and decision making.

For a utility with multiple correlated projects, the number of options at a decision point represents the enumerated combinations of the options for each project. For example, if a utility has 2 projects and each project has 6 options, then the total number of options is 36.

Since we will be sequentially making decisions over options, we will utilize a backward dynamic programming formulation. By maximizing the value of the investment that includes two projects during the planning horizon, we will determine the optimal sequential decisions over the options. We note that the backward dynamic programming formulation is subject to constraints such as budget and demand.

This chapter is organized as follows. Section 4.2 presents the GBM processes that represent the movements of the market values of two completed generation units. Section 4.3 shows how to approximate a GBM process by a binomial lattice model. Section 4.4 shows how to approximate the combination of two correlated GBM processes by a four-branch lattice model. Section 4.5 provides the values of options. Section 4.6 investigates the roles of the correlation coefficient. Section 4.7 provides the dynamic programming

formulation. An illustrative example is presented for managerial insights and economic implications in Section 4.8. Finally, concluding remarks are provided in Section 4.9.

4.2 Geometric Brownian Motion

Market conditions can be represented by the market value of completed generation units (see e.g., [48], [49]). For example, if the market value of completed generation units decreases dramatically in a period, then deferring may be a good option in the next period. However, at the beginning of the planning horizon when the strategic planning is made for a generation planning project, the generation unit that the project represents does not exist yet. To estimate the market value of a completed generation unit at the beginning of the planning horizon, we may use a currently existing generation unit that has similar features such as capacity and construction costs (see e.g., [40], [48]).

The market value of a completed generation unit is considered as the major source of uncertainties after the beginning of the planning horizon. Let X_i be the market value of the completed generation unit i . According to Teisberg [48], [49], it is reasonable to assume that X_i evolves stochastically over time according to a geometric Brownian motion (GBM) shown in the following equation.

$$dX_i = \mu_i X_i dt + \sigma_i X_i dz_i$$

where dz_i is the differential of a standard Wiener process (with mean 0 and variance dt) for the completed generation unit i . μ_i is the instantaneous expected rate of return on the completed generation unit i . σ_i is the instantaneous standard deviation of the rate of return on the completed generation unit i .

In this chapter, we consider a utility that has two interrelated generation planning projects. In other words, we assume that a utility has projects 1 and 2. dz_1 and dz_2 have a correlation coefficient ρ .

4.3 Binomial Lattice Model

Instead of developing complicated differential equations that may not have analytic solutions, we approximate the continuous GBMs by discrete binomial lattice models. In a binomial lattice model, the change in the value of X_i ($i = 1, 2$) has two possibilities: going up or going down.

These two possible values are defined as the multiples of the value of X_i in the previous period as shown in Figure 4.1 – a multiple u_i for up ($u_i > 1$) and a multiple d_i for down ($d_i < 1$), and $u_i d_i = 1$. The probabilities for up and down are p_u and $1 - p_u$, respectively. We note that the index of periods in Figure 4.1 represents the beginning of a period.

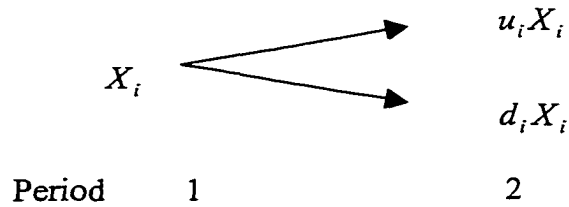


Figure 4.1 Binomial lattice model

Approximating a GBM by a binomial lattice model requires two steps. The first step is to transform the GBM for X_i to a process for $\ln X_i$. The second step is to find the suitable values for u_i, d_i , and p_{u_i} by matching the binomial lattice model to the process for $\ln X_i$.

A. Transformation

Let $W_i = \ln X_i$, then, we keep terms in dW_i up to first order in dt as in equation (4.1).

$$\begin{aligned} dW_i &= \frac{dW_i}{dX_i} dX_i + \frac{1}{2} \frac{d^2 W_i}{dX_i^2} (dX_i)^2 \\ &= \frac{1}{X_i} (\mu_i X_i dt + \sigma_i X_i dz_i) \\ &\quad - \frac{1}{2} \frac{1}{X_i^2} (\sigma_i^2 X_i^2 dt) \end{aligned} \tag{4.1}$$

It can be shown that $(dz_i)^2$ is equal to dt . We note that the only nonzero term in $(dX_i)^2$ is $\sigma_i^2 X_i^2 dt$. Therefore, we have

$$d \ln X_i = \left(\mu_i - \frac{1}{2} \sigma_i^2 \right) dt + \sigma_i dz_i \tag{4.2}$$

B. Matching

We match the expected value of the change in $\ln X_i$ in the binomial lattice model to the expected value of $d \ln X_i$ in equation (4.2). Also, we match the variance of the change in $\ln X_i$ in the binomial lattice model to the variance of $d \ln X_i$ in equation (4.2).

Let $X_i(t + \Delta t)$ be the value of X_i at time $t + \Delta t$. $\ln X_i(t + \Delta t)$ in the binomial lattice model can be expressed as

$$\ln X_i(t + \Delta t) = \ln X_i(t) + Y_i$$

where

$$Y_i = \begin{cases} \ln u_i & \text{with probability } p_{u_i} \\ \ln d_i & \text{with probability } 1 - p_{u_i} \end{cases}$$

Therefore, the matching is as follows:

$$\begin{aligned} E[Y_i] &= p_{u_i} \ln u_i + (1 - p_{u_i}) \ln d_i \\ &= (\mu_i - \frac{1}{2} \sigma_i^2) \Delta t \\ \text{var}(Y_i) &= E[Y_i^2] - (E[Y_i])^2 \\ &= (\ln u_i)^2 4p_{u_i} (1 - p_{u_i}) = \sigma_i^2 \Delta t \end{aligned} \quad (4.3)$$

The suitable values for u_i , d_i , and p_{u_i} are solved as [32]

$$p_{u_i} = \frac{1}{2} + \frac{1}{2} \left(\frac{\mu_i - \frac{1}{2} \sigma_i^2}{\sigma_i} \right) \sqrt{\Delta t} \quad (4.4)$$

$$u_i = e^{\sigma_i \sqrt{\Delta t}} \quad (4.5)$$

$$d_i = e^{-\sigma_i \sqrt{\Delta t}} \quad (4.6)$$

We note that by replacing μ_i by r , risk-free rate of return, p_{u_i} becomes risk-neutral probability.

After binomial lattice models for individual generation units are constructed, we now turn to constructing a discrete lattice model for the market value of two completed generation units that has a correlation coefficient ρ .

4.4 Four-Branch Lattice Model

The combination of two binomial lattices produces a four-branch lattice, namely, the branch with both X_1 and X_2 going up, the branch with X_1 going up and X_2 going down, the branch with X_1 going down and X_2 going up, and the branch with both X_1 and X_2 going down. If $\rho = 0$, then the joint probability for each branch is equal to the product of marginal probabilities.

If $\rho \neq 0$, the adjustments a_1 , a_2 , a_3 , and a_4 are needed to be added to the products of marginal probabilities. In Figure 4.2, the probabilities for the branches from top to bottom are therefore $p_{u_1} p_{u_2} + a_1$, $p_{u_1} (1 - p_{u_2}) + a_2$, $(1 - p_{u_1}) p_{u_2} + a_3$, and $(1 - p_{u_1})(1 - p_{u_2}) + a_4$, respectively.

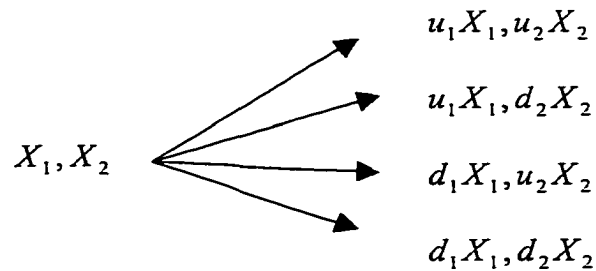


Figure 4.2 Four-branch lattice model

A. Transformation

Similarly, two steps are required to determine the suitable values for the adjustments. The first step is to develop a process for $\ln X_1 + \ln X_2$. It can be shown that the process is

$$d(\ln X_1 + \ln X_2) = (\mu_1 + \mu_2 - \frac{1}{2}\sigma_1^2 - \frac{1}{2}\sigma_2^2)dt + \sqrt{\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2} dz_3 \quad (4.7)$$

where dz_3 is the differential of a standard Wiener process which is different from dz_1 and dz_2 (see e.g., [40]).

B. Matching

The second step is to find the suitable values for the adjustments by matching the four-branch lattice model to the process in equation (4.7). By employing the values obtained in equation (4.4), equation (4.5), and equation (4.6), the expected value of the change in $\ln X_1 + \ln X_2$ in the four-branch lattice model is matched to the expected value of $d(\ln X_1 + \ln X_2)$ in equation (4.7). Therefore, we only need to match the variance of the change in $\ln X_1 + \ln X_2$ in the four-branch lattice model to the variance of $d(\ln X_1 + \ln X_2)$ in equation (4.7).

The matching is as follows:

$$\begin{aligned} \text{var}(Y_1 + Y_2) &= \text{var}(Y_1) + \text{var}(Y_2) + 2 \text{cov}(Y_1, Y_2) \\ &= \sigma_1^2 \Delta t + \sigma_2^2 \Delta t + 2\rho\sigma_1\sigma_2 \Delta t \end{aligned}$$

Since $\text{var}(Y_1)$ and $\text{var}(Y_2)$ are matched in equation (4.3), we only need to match the covariance term to determine the suitable values for the adjustments. a_1 , a_2 , a_3 , and a_4 are calculated as $\rho/4$, $-\rho/4$, $-\rho/4$, and $\rho/4$, respectively.

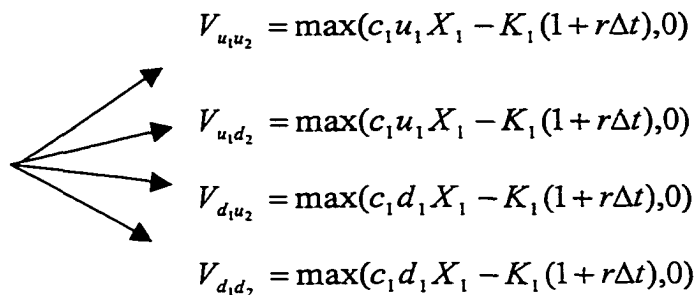
Market conditions determine which option should be exercised as well as the values of the options. After we construct a four-branch lattice model for the market value of two correlated generation units, in the next section, we present the values of the options in generation planning.

4.5 Values of Options

A. Call options

The options to construct a generation unit and to expand the constructed capacity level can be regarded as call options because the utility has the right, but not the obligation to make investment expenditures and receive the assets. For the option to construct the generation unit, the utility will exercise this option only if the market value of the completed generation unit exceeds the construction costs. For the option to expand the constructed capacity level, the utility will exercise this option only if the extra revenue after the expansion exceeds the expansion cost.

Suppose the utility has two existing generation units. Let us consider the option to expand the constructed capacity level of generation unit 1 at the beginning of period 2. Let K_1 denote the expansion cost at the beginning of period 1. Let c_1 denote the extra percentage of the market value of the generation unit 1 that the expansion generates. The value of the option at the beginning of period 2 is shown in Figure 4.3.



$$\begin{aligned}
 V_{u_1 u_2} &= \max(c_1 u_1 X_1 - K_1(1 + r\Delta t), 0) \\
 V_{u_1 d_2} &= \max(c_1 u_1 X_1 - K_1(1 + r\Delta t), 0) \\
 V_{d_1 u_2} &= \max(c_1 d_1 X_1 - K_1(1 + r\Delta t), 0) \\
 V_{d_1 d_2} &= \max(c_1 d_1 X_1 - K_1(1 + r\Delta t), 0)
 \end{aligned}$$

Figure 4.3 Value of the option to expand generation unit 1

In Figure 4.3, V 's represent the values of the option to expand the constructed capacity level of generation unit 1 under specific market conditions and Δt represents the length of a period.

Evaluating other call options requires changes in V 's. For example, for the option to expand the constructed capacity levels of two generation units, $V_{u_1u_2}$ is equal to

$$\max(c_1u_1X_1 - K_1(1+r\Delta t) + c_2u_2X_2 - K_2(1+r\Delta t), 0).$$

Then, the value of any option at the beginning of period 1 with respect to risk-neutral probabilities, V_0 , is equal to

$$\begin{aligned} V_0 = & (V_{u_1u_2} (p_{u_1} p_{u_2} + \frac{\rho}{4}) + V_{u_1d_2} (p_{u_1} p_{d_2} - \frac{\rho}{4}) \\ & + V_{d_1u_2} (p_{d_1} p_{u_2} - \frac{\rho}{4}) + V_{d_1d_2} (p_{d_1} p_{d_2} + \frac{\rho}{4})) / (1+r) \end{aligned} \quad (4.8)$$

where $p_{d_1} = 1 - p_{u_1}$, $p_{d_2} = 1 - p_{u_2}$.

B. Put options

The options to reduce the constructed capacity level of a generation unit can be considered as put options because the utility has the right, but not the obligation to give up some generation assets (e.g., sell some wind turbines for a wind generation unit) and save some generation costs. Similarly, a utility will exercise this option only if the money saved from reducing the capacity exceeds the amount of foregone revenue due to the capacity reduction.

For the option to reduce the constructed capacity level of generation unit 1 at the beginning of period 2, $V_{u_1u_2}$ is equal to $\max(G_1(1+r\Delta t) - c_1u_1X_1, 0)$, where G_1 is the

generation costs saved from the capacity reduction at the beginning of period 1 and c_1 is the reduced percentage of the market value of generation unit 1.

4.6 Role of Correlation Coefficient

In this section, we examine the role of correlation coefficient, ρ , in decision making (i.e., the decision to exercise an option at the beginning of period 1 or 2) and in the values of options in a two-period model.

The numerator of equation (4.8) can be expressed as

$$(V_{u_1u_2} p_{u_1} p_{u_2} + V_{u_1d_2} p_{u_1} p_{d_2} + V_{d_1u_2} p_{d_1} p_{u_2} + V_{d_1d_2} p_{d_1} p_{d_2}) + \frac{\rho}{4} (V_{u_1u_2} + V_{d_1d_2} - V_{u_1d_2} - V_{d_1u_2})$$

This implies the following conditions:

Condition 1: If $V_{u_1u_2} + V_{d_1d_2} = V_{u_1d_2} + V_{d_1u_2}$,

then ρ does not have role in the value of the option.

Condition 2: If $V_{u_1u_2} + V_{d_1d_2} > V_{u_1d_2} + V_{d_1u_2}$,

then the increase in the value of ρ will increase the value of the option.

Condition 3: If $V_{u_1u_2} + V_{d_1d_2} < V_{u_1d_2} + V_{d_1u_2}$,

then the increase in the value of ρ will decrease the value of the option.

If an option involves activities for only one generation unit, then condition 1 is satisfied. In other words, ρ does not have role in the value of the option. Moreover, in this case, ρ does not have role in decision making.

If an option involves activities for two generation units, then one of conditions 1, 2, and 3 is satisfied. In other words, ρ may have role in the value of the option. Now, if the option can be exercised at the beginning of period 1 only or 2 only, then ρ does not have a role in decision making. All these observation can be verified mathematically. For example, if the option can be exercised either at the beginning of period 1 or at the beginning of period 2, then ρ will have a role in decision making if condition 2 or condition 3 is satisfied.

4.6.1 Numerical Example

In this example, we consider an option to reduce the constructed capacity levels of two generation units. This option can be exercised either at the beginning of period 1 or at the beginning of period 2. We want to determine when to exercise this option and the corresponding values of the option.

The length of a period is 3 months. The hypothetical data are as follows:

$r = 6\%$ (annual), $\mu_1 = 30\%$ (annual), $\sigma_1 = 60\%$ (annual), $\mu_2 = 40\%$ (annual),
 $\sigma_2 = 40\%$ (annual), $c_1 = c_2 = 10\%$, $G_1 = \$70$ million, $X_1 = \$500$ million, $G_2 = \$90$ million,
 $X_2 = \$800$ million.

Then, we use equation (4.4), equation (4.5), and equation (4.6) to obtain $p_{u_1} = 0.45$,
 $u_1 = 1.35$, $d_1 = 0.74$ $p_{u_2} = 0.4875$, $u_2 = 1.22$, $d_2 = 0.82$.

For the option that can be exercised at the beginning of period 2, the values at the beginning of period 2 are calculated as follows: $V_{u_1 u_2} = 0$, $V_{u_1 d_2} = 29.3$, $V_{d_1 u_2} = 27.8$,
 $V_{d_1 d_2} = 59.8$. Therefore, the value of this option at the beginning of period 1 with respect to risk-neutral probabilities is

$$\begin{aligned}
V_0 &= (29.3*(0.45*0.5125 - \rho/4) \\
&\quad + 27.8*(0.55*0.4875 - \rho/4) \\
&\quad + 59.8*(0.55*0.5125 + \rho/4)) / (1+r) \\
&= 30.61 + 0.665\rho
\end{aligned}$$

For the option that can be exercised at the beginning of period 1, the value at the beginning of period 1 is $\max(G_1 - c_1X_1 + G_2 - c_2X_2, 0) = 30$. We compare $30.61 + 0.665\rho$ with 30. If $\rho < -0.917$, then we should exercise the option at the beginning of period 1 and the value of the option is \$30 million. If $\rho > -0.917$, then we should exercise the option at the beginning of period 2 and the value of the option is $\$30.61 + 0.665\rho$ million.

4.7 Dynamic Programming

Because the selection of an option in one period will affect the selection of options in subsequent periods, the use of backward dynamic programming is necessary. We solve the generation planning problem recursively, starting from the end and moving back to determine the optimal sequential decisions for a utility and the corresponding value of the investment.

Corresponding to the four-branch lattice for the market values of the completed generation units, each option has a four-branch lattice representing net cash flows that the option would generate if it is exercised. Each net cash flow represents the payoff under a specific market condition in a specific period. For some options, the net cash flows are set at negative infinity in some periods. For example, if both the construction and expansion of a generation planning project require one period to complete, then for that project the earliest

feasible net cash flows for the option to operate at the expanded capacity level occur in period 3. Therefore, the net cash flows are set at negative infinity in periods 1 and 2.

The value of the investment at the beginning of t , given state s is newly observed while option during $t-1$ was m , can be obtained from the expected value of the investment at the beginning of $t+1$ as follows:

$$V_t^s(m) = \max_i \left(C_t^s(i) + \frac{E[V_{t+1}(i)]}{1+r} - I(m \rightarrow i) \right)$$

subject to

$$I(m \rightarrow i) = 0 \quad \text{for } i = m$$

$$i \in F_t^s$$

where

s = index of states (the changes in market value of the complete projects, i.e., (up, up), (up, down), (down, up), (down, down))

i = index of options

t = index of periods, $t=1, \dots, T-1$, where T is the last period under planning

$V_t^s(m)$ = value of the investment at the beginning of t , given state s is newly observed while operating in option m (assume optimal future decisions)

$C_t^s(i)$ = net cash flow at the beginning of t , given state s is newly observed while operating in option i

$E[V_{t+1}(i)]$ = expected value of $V_{t+1}(i)$ with respect to risk-neutral probabilities

$V_{t+1}(i)$ = value of the investment at the beginning of $t+1$ while operating in option i (a random variable)

r = risk-free rate of return in a period

$I(m \rightarrow i)$ = switching cost from option m to option i

F_t^s = set of feasible options at the beginning of t , given state s is newly observed

For the case of two correlated generation planning projects (i.e., a four-branch lattice model), the probability distribution for $V_{t+1}(i)$ is as follows:

$$V_{t+1}(i) = \begin{aligned} & V_{t+1}^{u_1 u_2}(i) \text{ with probability } p_{u_1} p_{u_2} + \frac{\rho}{4} \\ & V_{t+1}^{u_1 d_2}(i) \text{ with probability } p_{u_1} p_{d_2} - \frac{\rho}{4} \\ & V_{t+1}^{d_1 u_2}(i) \text{ with probability } p_{d_1} p_{u_2} - \frac{\rho}{4} \\ & V_{t+1}^{d_1 d_2}(i) \text{ with probability } p_{d_1} p_{d_2} + \frac{\rho}{4} \end{aligned}$$

We note that the above probabilities are risk-neutral probabilities. With respect to the risk-neutral probabilities, we discount $E[V_{t+1}(i)]$ at the risk-free rate of return.

Switching from an option to another option requires a switching cost. Some switches are not possible. For example, switching from operating at a constructed capacity level to deferring construction for the same project is not possible. In this case, the switching cost is assumed to be infinite.

We note that each option, given a state and a period, should be checked for feasibility. That is, there may be various constraints (such as financial or demand constraints) that make it impossible to exercise an option. For example, a utility may not have enough budget to expand two generation units' capacity at the same time. Therefore, such an option is not feasible to switch to. Similarly, a utility may need a certain amount of capacity to serve customers at a specific time. Therefore, the option to defer the construction of two projects may not be feasible at that time point.

With the switching costs and constraints, the number of the feasible options that should be evaluated over the planning horizon will be greatly reduced.

Let T denote the last period under planning. The value of the investment at the beginning of T , given state s is newly observed while option during $T-1$ was m , is as follows:

$$V_T^s(m) = \max_i (C_T^s(i) - I(m \rightarrow i))$$

subject to

$$I(m \rightarrow i) = 0 \quad \text{for } i = m$$

$$i \in F_T^s$$

4.8 Numerical Example

4.8.1 Input Data

We now provide a numerical example with hypothetical data to illustrate the main features of our model. In this example, there are two generation planning projects (1 and 2). Project 1 represents a potential wind power generation unit (combination of wind turbines). Project 2 represents a potential gas turbine generation unit. Each project has the options to defer the construction of the generation unit, to construct the generation unit, to operate at the constructed capacity level, to expand the constructed capacity level, to operate at the expanded capacity level, to sell the constructed generation unit at its salvage value, and to sell the expanded generation unit at its salvage value.

The planning horizon is 3 periods and each period is one year. The 3-period four-branch lattice for the market values of the two completed generation units is shown in Figure 4.4. In Figure 4.4, the index of periods represents the beginning of a period. We assume that

both construction and expansion of both generation units require one period to complete. Risk-free rate of return, r , is assumed to be 0.06. The data for the generation units that the two generation planning projects represent is listed in Table 4.1. The correlation coefficient between the market values of the two completed generation units is assumed to be -0.5 .

From Table 4.1, we can calculate the risk-neutral probabilities of the binomial lattice models for individual projects as $p_{u_1} = 0.6$, $p_{d_1} = 0.4$, $p_{u_2} = 0.475$, $p_{d_2} = 0.525$. Then, risk-neutral probabilities of the four-branch lattice model for two correlated projects can be calculated as 0.16, 0.44, 0.315, and 0.085 for the branch from top to bottom for each node in Figure 4.4, respectively.

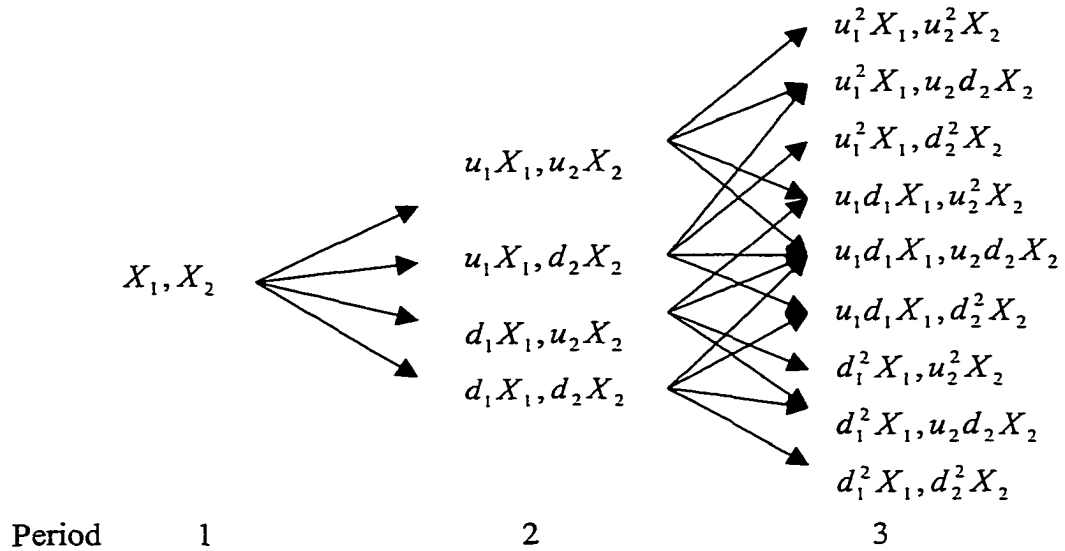


Figure 4.4 3-period four-branch lattice

Table 4.1 Project data

	Project 1	Project 2
Annual rate of return on the completed unit	0.2	0.3
Annual standard deviation of the completed unit	0.2	0.4
Constructed capacity (MW)	140	150
Capacity after expansion (MW)	180	200

As stated previously, each project has 7 options at each decision time point (i.e., node). Since there are two projects, at a node, a combination pair of options can be formed with one option for each project. Therefore, the total number of combination options at each node is 49. For each pair of combination options, we have a 3-period 4-branch lattice depicted in Figure 4.4. We note that at each node, we are newly informed of the specific market values of the generation units. Given the specific market values, we can obtain the net cash flow ($C_t^s(i)$). Since there are 14 nodes (1+4+9) for 49 combination pairs of options, we calculate $14 \times 49 = 686$ $C_t^s(i)$'s. The complete numerical data are available upon request.

At $t = 3$, we assume that there are capacity constraints requiring 350 MW of capacity for the first 8 nodes and 345 MW of capacity for the last node ($d_1^2 X_1, d_2^2 X_2$) in Figure 4.4. At $t = 2$, we assume that there are capacity constraints requiring 200 MW of capacity for the first 3 nodes and 180 MW of capacity for the last node ($d_1 X_1, d_2 X_2$). At $t = 1$, we assume that there are no constraints.

In this example, some of switches are logically impossible. For example, for the same project, we can not switch from deferring to operating, from deferring to expanding, and from deferring to selling. For impossible switches, the switch costs are assumed to be infinite. In this example, all the switching costs for possible switches are assumed to be zero.

4.8.2 *Solution*

At all nodes at $t = 3$, the only option that satisfies the capacity constraints is the option to operate both units at the expanded capacity level. At all nodes at $t = 2$, there are four options satisfying the capacity constraints. They are the option to operate both units at the constructed capacity level, the option to operate unit 1 at the constructed capacity and expand unit 2, the option to expand unit 1 and operate unit 2 at the constructed capacity level, and expand both units. Since at $t = 2$ all the feasible options are in operating conditions, the utility must construct both units at $t = 1$.

After we examine constraints and obtain feasible options for all nodes, we examine the switching costs. There are four feasible options at $t = 2$ and one feasible option at $t = 3$. However, at $t = 2$ the option to expand both units is the only option that does not incur infinite switching cost. Therefore, the option to expand both units is the optimal option at $t = 2$.

In summary, the optimal options are to construct both units at $t = 1$, to expand both units at $t = 2$, and to operate both units at the expanded capacity level at $t = 3$. The corresponding optimal investment value at the beginning of decision time point 1 (with the assumption that generation units 1 and 2 are deferred from decision time point 0) is calculated as \$ 36.5 millions.

To reduce the number of options that should be considered in dynamic programming, we propose a two-step procedure via a numerical example. First, we eliminate the options that violate the constraints. Secondly, starting from the last period, we ignore the options that incur infinite switching costs. Via this procedure, the size of the dynamic programming can be greatly reduced.

4.9 Concluding Remarks

In this chapter, we developed a generation planning model for two correlated projects based on the real options theory. Specifically, first, we constructed a four-branch lattice model that approximates the combination of two correlated GBM processes for two correlated projects. Then, we investigated the roles of the correlation coefficient in both the value of an option and decision making. Finally, by maximizing the value of investment that includes two correlated projects, we showed how the optimal sequential decisions over the options can be determined.

The contributions of this chapter are as follows. First, we constructed a four-branch lattice model that explicitly incorporates the correlation coefficient of two projects. Second, we showed that the value of the correlation coefficient may affect both decision making and the value of an option.

As an interesting future research, we plan to study intuitive reasons for the role (or the lack of the role) of ρ in the value of an option and decision making.

CHAPTER 5. A MULTIPLE-OPTION MODEL WITH MULTIPLE CORRELATED PROJECTS FOR GENERATION PLANNING

5.1 Introduction

In Chapter 4, for two correlated projects, we designed and analyzed a four-branch lattice model based on the real options approach. In this chapter, we extend the model in Chapter 4 by considering a general case of n ($n \geq 2$) correlated projects.

In order to develop a model for multiple correlated projects, we first extend the four-branch lattice model for two correlated projects to an eight-branch lattice model for three correlated projects. Namely, project 1, 2, and 3. These three correlated projects will have the following sub-relationship. i.e., the correlation between projects 1 and 2, the correlation between projects 2 and 3, and the correlation between projects 1 and 3. Each of these correlations can be represented by a four-branch lattice. With three projects, we will need an eight-branch lattice (representing the up and down of the market value of each project). As for the risk-neutral probabilities of the eight-branch lattice model, we obtain them by solving a system of equations generated by the risk-neutral probabilities of three four-branch lattice models.

Once our lattice model for multiple correlated projects is constructed, we proceed to compare our model with other lattice models. Even though there are very few papers for correlated capital investment projects (see e.g., [15]), there do exist studies on how correlated geometric Brownian motion processes are approximated by discrete lattices (see e.g., [12], [18], [25]).

Next, we investigate the roles of correlation coefficients by using a two-period lattice model. For this investigation, we first examine the case of three correlated projects. Next, we examine the general case of n ($n \geq 2$) correlated projects. Based on explicit sets of conditions, the roles of correlation coefficients are presented.

Finally, we provide a dynamic programming model for sequential decision making purposes. Via an example with three correlated projects, we illustrate how the optimal decisions over options for multiple correlated projects can be determined.

The organization of this chapter is as follows. In Section 5.2, we provide an eight-branch lattice model for three correlated projects. In Section 5.3, we extend the eight-branch lattice to a lattice model for multiple correlated projects. Section 5.4 compares our approximation model with other approximation models. Section 5.5 investigates the roles of correlation coefficients. Section 5.6 provides a dynamic programming model for sequential decision making. This will be followed by an illustrative example with three correlated projects in Section 5.7. Finally, concluding remarks and comments on future research are provided in Section 5.8.

5.2 Lattice Model for Three Correlated Projects

A utility may consider strategic options for multiple correlated projects where each project represents a power plant. The market value of each power plant is assumed to evolve stochastically according to a geometric Brownian motion (GBM) process and is correlated to the market values of other power plants. Thus, there are multiple correlated GBM processes for the market values of power plants, which are considered as the underlying assets for the strategic options for multiple correlated projects.

To estimate the values of such options, we construct lattice models by matching the key parameters such as means, variances, and covariances to the correlated GBM processes. Our first attempt is to develop a lattice model for three correlated projects. Then, we will generalize this lattice for n correlated projects ($n \geq 2$).

Let X_i ($i = 1, 2, 3$) be the market value of power plant i and it evolves according to:

$$dX_i = \mu_i X_i dt + \sigma_i X_i dz_i$$

where dz_1 and dz_2 has correlation coefficient ρ_{12} , dz_1 and dz_3 has correlation coefficient ρ_{13} , dz_2 and dz_3 has correlation coefficient ρ_{23} .

One GBM process is approximated by a binomial lattice and the combination of two GBM processes is approximated by a four-branch lattice. Similarly, the combination of three GBM processes can be approximated by an eight-branch lattice as shown in Figure 5.1. For example, the first branch in Figure 5.1 represents that all of X_1 , X_2 , X_3 move up. Let p_{u_i} denote the risk-neutral probability for the up-move in a binomial lattice for underlying asset i . If three GBM processes are not correlated, then the joint probability for each branch in Figure 5.1 is equal to the product of three marginal probabilities. For example, the joint probability for the first branch in Figure 5.1 is therefore equal to $\prod_{i=1}^3 p_{u_i}$. Otherwise, an adjustment, a_k , $k = 1, \dots, 8$, is needed to be added to the product of three marginal probabilities in branch k . For example, the joint probability for the first branch in Figure 5.1 is therefore equal to $\prod_{i=1}^3 p_{u_i} + a_1$. Now, we need to determine the values of a_k s.

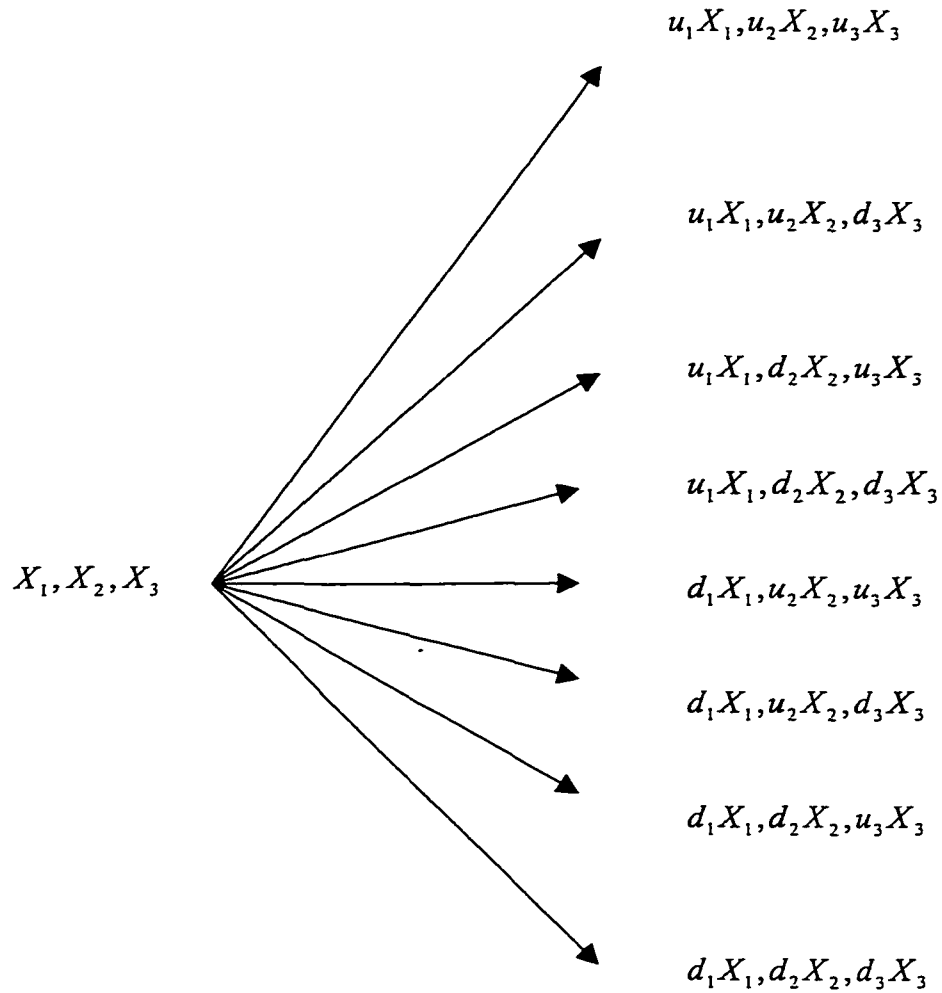


Figure 5.1 Eight-branch lattice for three underlying assets

A binomial lattice for one underlying asset is constructed by matching the mean and the variance of the lattice to the mean and the variance of the GBM process. Then, by using two matched binomial lattices for two individual underlying assets, a four-branch lattice for two correlated underlying assets is constructed by matching the covariance of the four-branch lattice to the covariance of the combination of the two GBM processes. For example, observe that the bottom branch of Figure 5.1 (d_1X_1, d_2X_2, d_3X_3) is from the combination of the bottom branches of the three four-branch lattices in Figure 5.2.

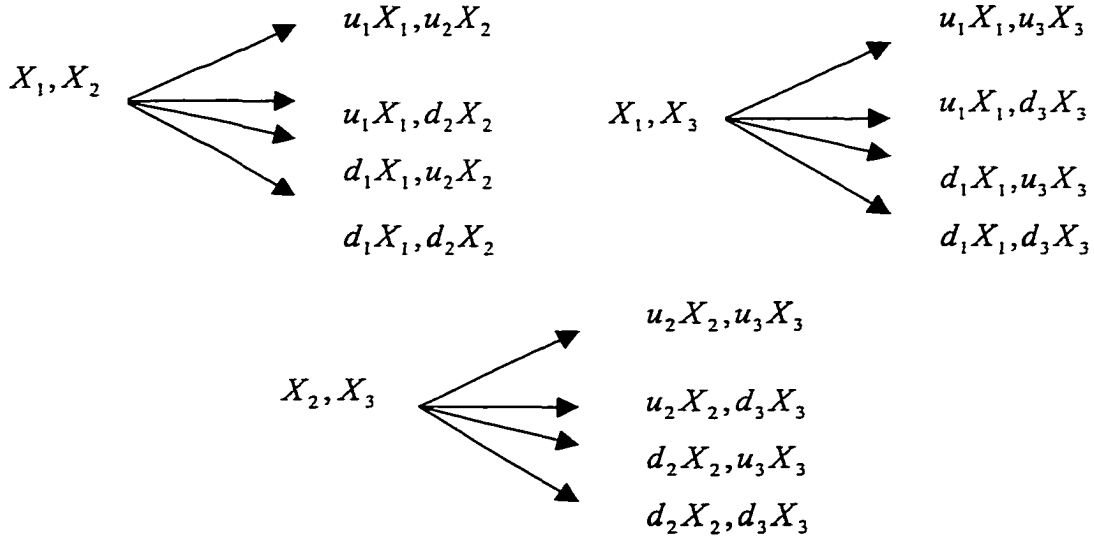


Figure 5.2 Four-branch lattices for three pairs of correlated underlying assets

The probabilities for the branches from top to bottom for the pair of underlying assets

i, j are $p_{u_i} p_{u_j} + \frac{\rho_{ij}}{4}$, $p_{u_i} p_{d_j} - \frac{\rho_{ij}}{4}$, $p_{d_i} p_{u_j} - \frac{\rho_{ij}}{4}$, $p_{d_i} p_{d_j} + \frac{\rho_{ij}}{4}$, respectively.

The adjustments as well as the probabilities for the branches in Figure 5.1 can be determined by using the four-branch lattices in Figure 5.2. Since key parameters such as means, variances, and covariances have already been matched, the eight-branch lattice based on four-branch lattices will automatically match the GBM processes for each individual underlying asset, as well as the combination of two correlated GBM processes for each pair of underlying assets.

Let us consider the pair of underlying assets 1 and 2. The sum of the probabilities associated with the first two branches (i.e., $u_1 X_1, u_2 X_2, u_3 X_3$ and $u_1 X_1, u_2 X_2, d_3 X_3$) in

Figure 5.1 should be equal to the probability associated with the first branch $(u_1 X_1, u_2 X_2)$ of the first lattice in Figure 5.2. Mathematically,

$$p_{u_1} p_{u_2} p_{u_3} + a_1 + p_{u_1} p_{u_2} p_{d_3} + a_2 = p_{u_1} p_{u_2} + \frac{\rho_{12}}{4}$$

This implies $a_1 + a_2 = \frac{\rho_{12}}{4}$. By the same logic, the following system of equations can be

derived to determine the values of a_k s.

$$a_1 + a_2 = \frac{\rho_{12}}{4}$$

$$a_3 + a_4 = -\frac{\rho_{12}}{4}$$

$$a_5 + a_6 = -\frac{\rho_{12}}{4}$$

$$a_7 + a_8 = \frac{\rho_{12}}{4}$$

$$a_1 + a_5 = \frac{\rho_{23}}{4}$$

$$a_2 + a_6 = -\frac{\rho_{23}}{4}$$

$$a_3 + a_7 = -\frac{\rho_{23}}{4}$$

$$a_4 + a_8 = \frac{\rho_{23}}{4}$$

$$a_1 + a_3 = \frac{\rho_{13}}{4}$$

$$a_2 + a_4 = -\frac{\rho_{13}}{4}$$

$$a_5 + a_7 = -\frac{\rho_{13}}{4}$$

$$a_6 + a_8 = \frac{\rho_{13}}{4}$$

This system of equations has rank 7 and there are 8 unknowns. Thus theoretically, the number of solutions is infinite. However, we can consider the following symmetric relations: $a_1 = a_8$, $a_2 = a_7$, $a_3 = a_6$, $a_4 = a_5$. i.e., the adjustments where market value directions of each project are opposite are the same. Under this symmetric relations, the 12 equations with 8 unknowns are reduced to the following 6 equations and 4 unknowns.

$$a_1 + a_2 = \frac{\rho_{12}}{4}$$

$$a_3 + a_4 = -\frac{\rho_{12}}{4}$$

$$a_1 + a_4 = \frac{\rho_{23}}{4}$$

$$a_2 + a_3 = -\frac{\rho_{23}}{4}$$

$$a_1 + a_3 = \frac{\rho_{13}}{4}$$

$$a_2 + a_4 = -\frac{\rho_{13}}{4}$$

Here the rank can be verified to be 4 and the following solution is unique:

$$a_1 = \frac{\rho_{12} + \rho_{23} + \rho_{13}}{8}, a_2 = \frac{\rho_{12} - \rho_{23} - \rho_{13}}{8}, a_3 = \frac{-\rho_{12} - \rho_{23} + \rho_{13}}{8}, a_4 = \frac{-\rho_{12} + \rho_{23} - \rho_{13}}{8},$$

$$a_5 = \frac{-\rho_{12} + \rho_{23} - \rho_{13}}{8}, \quad a_6 = \frac{-\rho_{12} - \rho_{23} + \rho_{13}}{8}, \quad a_7 = \frac{\rho_{12} - \rho_{23} - \rho_{13}}{8},$$

$$a_8 = \frac{\rho_{12} + \rho_{23} + \rho_{13}}{8}.$$

We note that the only lattices that need to be approximated to the GBM processes are binomial lattices for individual underlying assets and four-branch lattices for two correlated underlying assets. After these lattices are constructed, an eight-branch lattice for three correlated underlying assets can be constructed by solving a system of equations based on four-branch lattices. Similarly, a sixteen-branch lattice for four correlated underlying assets can be constructed by solving a system of equations based on eight-branch lattices.

5.3 Lattice Model for N Correlated Projects

By the symmetry of the expressions obtained for four-branch and eight-branch lattices, the probability associated with branch k of the lattice for n correlated underlying assets is

$$p_k = \prod_{i=1}^n p_{\delta_i(k)} + \frac{1}{2^n} \sum_{i=1}^n \sum_{\substack{j=2 \\ i < j}}^n \delta_{ij}(k) \rho_{ij} \quad k = 1, \dots, 2^n$$

where

$$\delta_i(k) = \begin{cases} u_i & \text{if asset } i \text{ has an up - move in branch } k \\ d_i & \text{if asset } i \text{ has a down - move in branch } k \end{cases}$$

$$\delta_{ij}(k) = \begin{cases} 1 & \text{if both asset } i \text{ and asset } j \text{ move in same direction in branch } k \\ -1 & \text{if both asset } i \text{ and asset } j \text{ move in opposite direction in branch } k \end{cases}$$

5.4 Comparisons with Other Approximation Models

Our lattice model for n correlated underlying assets is constructed based on binomial lattice models for individual underlying assets. A binomial lattice model is constructed by choosing the suitable values for the key parameters in the lattice (Cox, Ross, and Rubinstein [17]). Specifically, suitable values for three unknowns are determined by solving a system of three equations. The three unknowns are the probability associated with an up-move, a multiple for an up-move, and a multiple for a down-move. The three equations are the equation to match the mean of the lattice to the mean of the GBM process, the equation to match the variance of the lattice to the variance of the GBM process, and the equation that imposes the assumption that the product of the two multiples is equal to one.

However, Cox, Ross, and Rubinstein approach (CRR) is not the only approach of constructing a binomial lattice. Instead of imposing the assumption that the product of the two multiples is equal to one, Hull [23] fixes the probability at 0.5. By solving two equations for two unknowns, Hull determines the multiple for an up-move as $u = e^{(r - \frac{\sigma^2}{2})\Delta t + \sigma\sqrt{\Delta t}}$ and the multiple for a down-move as $d = e^{(r - \frac{\sigma^2}{2})\Delta t - \sigma\sqrt{\Delta t}}$. Hull claims that the advantage of his model over the CRR approach is that the probabilities are always 0.5. In contrast, the CRR approach may give negative probabilities if the length of a period (Δt) is sufficiently large. Furthermore, Hull claims the disadvantage of his model is that it is not easy to determine the hedging strategies because the value of the underlying asset does not recombine after two periods.

The CRR approach has been extended to four-branch lattices for two underlying assets by Boyle et al. [13] and Wang and Min [57]. Boyle et al. equate the characteristic

function of the four-branch lattice with the characteristic function of the combination of two GBM processes.

The difference between Boyle's four-branch model and Wang and Min's four-branch model is in risk-neutral probabilities. In Boyle's model, the probabilities (e.g., the probability of (up, up)) keep terms up to $\sqrt{\Delta t}$. In Wang and Min's model, the probabilities keep terms up to Δt (i.e., this can be observed by expanding the mathematical expressions).

5.5 Roles of Correlation Coefficients

Let us consider an option for a utility with three correlated projects in a two-period model. In this model, the evolution of the three underlying assets is represented by an eight-branch lattice. Suppose the option can be exercised at the beginning of period 2. Let $V_{u_1 u_2 u_3}$ denote the value of the option at the beginning of period 2 under the market condition that all three underlying assets just moved up. For example, if the option is to expand the existing capacity levels of power plants 1, 2, and 3, then

$$V_{u_1 u_2 u_3} = \max\left(\sum_{i=1}^3 (c_i u_i X_i - K_i(1+r)), 0\right)$$

where

c_i = extra percentage of the market value of power plant i that the expansion generates

u_i = multiple for an up-move for power plant i

K_i = expansion cost for power plant i at the beginning of period 1

r = risk-free rate of return in a period

Then, the value of the option that can be exercised at the beginning of period 2 at the beginning of period 1 with respect to risk-neutral probabilities, V_0 , is equal to:

$$\begin{aligned}
V_0 = & \frac{1}{1+r} (V_{u_1u_2u_3} (p_{u_1} p_{u_2} p_{u_3} + \frac{\rho_{12} + \rho_{13} + \rho_{23}}{8}) + V_{u_1u_2d_3} (p_{u_1} p_{u_2} p_{d_3} + \frac{\rho_{12} - \rho_{13} - \rho_{23}}{8}) \\
& + V_{u_1d_2u_3} (p_{u_1} p_{d_2} p_{u_3} + \frac{-\rho_{12} + \rho_{13} - \rho_{23}}{8}) + V_{u_1d_2d_3} (p_{u_1} p_{d_2} p_{d_3} + \frac{-\rho_{12} - \rho_{13} + \rho_{23}}{8}) \\
& + V_{d_1u_2u_3} (p_{d_1} p_{u_2} p_{u_3} + \frac{-\rho_{12} - \rho_{13} + \rho_{23}}{8}) + V_{d_1u_2d_3} (p_{d_1} p_{u_2} p_{d_3} + \frac{-\rho_{12} + \rho_{13} - \rho_{23}}{8}) \\
& + V_{d_1d_2u_3} (p_{d_1} p_{d_2} p_{u_3} + \frac{\rho_{12} - \rho_{13} - \rho_{23}}{8}) + V_{d_1d_2d_3} (p_{d_1} p_{d_2} p_{d_3} + \frac{\rho_{12} + \rho_{13} + \rho_{23}}{8}))
\end{aligned}$$

To determine the roles of the correlation coefficients in the value of the option, we calculate the partial derivatives of V_0 with respect to ρ_{12} , ρ_{13} , and ρ_{23} as follows:

$$\frac{\partial V_0}{\partial \rho_{12}} = \frac{1}{8(1+r)} (V_{u_1u_2u_3} + V_{u_1u_2d_3} - V_{u_1d_2u_3} - V_{u_1d_2d_3} - V_{d_1u_2u_3} - V_{d_1u_2d_3} + V_{d_1d_2u_3} + V_{d_1d_2d_3})$$

$$\frac{\partial V_0}{\partial \rho_{13}} = \frac{1}{8(1+r)} (V_{u_1u_2u_3} - V_{u_1u_2d_3} + V_{u_1d_2u_3} - V_{u_1d_2d_3} - V_{d_1u_2u_3} + V_{d_1u_2d_3} - V_{d_1d_2u_3} + V_{d_1d_2d_3})$$

$$\frac{\partial V_0}{\partial \rho_{23}} = \frac{1}{8(1+r)} (V_{u_1u_2u_3} - V_{u_1u_2d_3} - V_{u_1d_2u_3} + V_{u_1d_2d_3} + V_{d_1u_2u_3} - V_{d_1u_2d_3} - V_{d_1d_2u_3} + V_{d_1d_2d_3})$$

These imply the following conditions:

Condition 1: If $V_{u_1u_2u_3} + V_{u_1u_2d_3} + V_{d_1d_2u_3} + V_{d_1d_2d_3} = V_{u_1d_2u_3} + V_{u_1d_2d_3} + V_{d_1u_2u_3} + V_{d_1u_2d_3}$,

then ρ_{12} has no role in the value of the option.

Condition 2: If $V_{u_1u_2u_3} + V_{u_1u_2d_3} + V_{d_1d_2u_3} + V_{d_1d_2d_3} > V_{u_1d_2u_3} + V_{u_1d_2d_3} + V_{d_1u_2u_3} + V_{d_1u_2d_3}$,

then the increase in the value of ρ_{12} will increase the value of the option.

Condition 3: If $V_{u_1u_2u_3} + V_{u_1u_2d_3} + V_{d_1d_2u_3} + V_{d_1d_2d_3} < V_{u_1d_2u_3} + V_{u_1d_2d_3} + V_{d_1u_2u_3} + V_{d_1u_2d_3}$,

then the increase in the value of ρ_{12} will decrease the value of the option.

Condition 4: If $V_{u_1u_2u_3} + V_{u_1d_2u_3} + V_{d_1u_2d_3} + V_{d_1d_2d_3} = V_{u_1u_2d_3} + V_{u_1d_2d_3} + V_{d_1u_2u_3} + V_{d_1d_2u_3}$,
then ρ_{13} has no role in the value of the option.

Condition 5: If $V_{u_1u_2u_3} + V_{u_1d_2u_3} + V_{d_1u_2d_3} + V_{d_1d_2d_3} > V_{u_1u_2d_3} + V_{u_1d_2d_3} + V_{d_1u_2u_3} + V_{d_1d_2u_3}$,
then the increase in the value of ρ_{13} will increase the value of the option.

Condition 6: If $V_{u_1u_2u_3} + V_{u_1d_2u_3} + V_{d_1u_2d_3} + V_{d_1d_2d_3} < V_{u_1u_2d_3} + V_{u_1d_2d_3} + V_{d_1u_2u_3} + V_{d_1d_2u_3}$,
then the increase in the value of ρ_{13} will decrease the value of the option.

Condition 7: If $V_{u_1u_2u_3} + V_{u_1d_2d_3} + V_{d_1u_2u_3} + V_{d_1d_2d_3} = V_{u_1u_2d_3} + V_{u_1d_2u_3} + V_{d_1u_2d_3} + V_{d_1d_2u_3}$,
then ρ_{23} has no role in the value of the option.

Condition 8: If $V_{u_1u_2u_3} + V_{u_1d_2d_3} + V_{d_1u_2u_3} + V_{d_1d_2d_3} > V_{u_1u_2d_3} + V_{u_1d_2u_3} + V_{d_1u_2d_3} + V_{d_1d_2u_3}$,
then the increase in the value of ρ_{23} will increase the value of the option.

Condition 9: If $V_{u_1u_2u_3} + V_{u_1d_2d_3} + V_{d_1u_2u_3} + V_{d_1d_2d_3} < V_{u_1u_2d_3} + V_{u_1d_2u_3} + V_{d_1u_2d_3} + V_{d_1d_2u_3}$,
then the increase in the value of ρ_{23} will decrease the value of the option.

If an option involves activities for only one power plant, then conditions 1, 4, and 7 are satisfied. In other words, ρ_{12} , ρ_{13} , and ρ_{23} have no roles in the value of the option. If an option involves activities for two power plants, for example, power plants 1 and 2, then ρ_{13} and ρ_{23} have no roles in the value of the option (see conditions 4 and 7). On the other hand, if an option involves activities for all three power plants, then ρ_{12} , ρ_{13} , and ρ_{23} may have roles in the value of the option (due to conditions 2, 3, 5, 6, 8, 9).

Now, let us consider an option for a utility with n correlated projects over a period where the evolution of the n underlying assets is represented by a 2^n -branch lattice. Then,

the number of correlation coefficients is $\frac{1}{2}n(n-1)$. Let V_k , $k = 1, 2, 3, \dots, 2^n$, denote the value of the option at the beginning of period 2 under the market condition k (branch k). For underlying assets f and g , ρ_{fg} has the following conditions:

Condition 1: If $\sum_{k: f, g \text{ move in same direction}} V_k = \sum_{k: f, g \text{ move in opposite direction}} V_k$, then ρ_{fg} has no role in the value of the option.

Condition 2: If $\sum_{k: f, g \text{ move in same direction}} V_k > \sum_{k: f, g \text{ move in opposite direction}} V_k$, then the increase in the value of ρ_{fg} will increase the value of the option.

Condition 3: If $\sum_{k: f, g \text{ move in same direction}} V_k < \sum_{k: f, g \text{ move in opposite direction}} V_k$, then the increase in the value of ρ_{fg} will decrease the value of the option.

If an option involves activities for only one power plant, then no correlation coefficient has any role in the value of the option. If an option involves activities for only two power plants (i.e., f and g), then ρ_{fg} may have a role in the value of the option.

Similarly, if an option involves activities for n' power plants, then the corresponding correlation coefficients of the n' power plants may have a role in the value of the options.

5.6 Sequential Strategic Options

Thus far, we investigated the role of correlation coefficients. In this section, we will utilize a dynamic programming model to formulate sequential decision making processes.

Specifically, we assume that strategic options can only be exercised at the beginning of a period, and there are n correlated projects. It can be shown that the number of market

conditions (states) in a multiple-period lattice at the beginning of period t is t^n where n is the number of projects. For example, if there are three projects, then the number of states at the beginning of period 1 is 1, the number of states at the beginning of period 2 is 8, and the number of states at the beginning of period 3 is 27. Figure 5.3 shows the evolution of three underlying assets in a three-period eight-branch lattice. The values of three underlying assets at the beginning of period 3 denoted by numbers in Figure 5.3 are listed in Table 5.1.

For a utility with multiple projects where each project has multiple options, the number of options at a decision point represents the enumerated combinations of the options for each project. For example, if a utility has 3 projects and each project has 4 options, then the total number of options are 64.

Since options are the derivative assets of underlying assets, each option is associated with a lattice that has the same number of states and the same probabilities for states as the lattice for underlying assets. Each option's lattice represents net cash flows that the option would generate if it is exercised. Each net cash flow represents the payoff under a specific market condition in a specific period.

Because the selection of an option in a period will affect the selection of options in subsequent periods, we apply backward dynamic programming to determine the optimal strategic options over the planning horizon. We solve this problem recursively, starting from the end and moving back to determine the optimal strategic options for a utility and the corresponding value of the investment. We note that the value of investment represents the sum of accumulated values of options for each project.

The value of the investment at the beginning of t , given state s is newly observed while option during $t-1$ was m , can be obtained from the sum of net cash flow ($C_t^s(i)$), the

discounted expected value of the investment at the beginning of $t+1$ ($\frac{E[V_{t+1}(i)]}{1+r}$), and the negative value of the switching cost ($I(m \rightarrow i)$) as follows:

$$V_t^s(m) = \max_i (C_t^s(i) + \frac{E[V_{t+1}(i)]}{1+r} - I(m \rightarrow i)) \quad (5.1)$$

s.t.

$$I(m \rightarrow i) = 0 \quad \text{for } i = m$$

$$i \in F_t^s$$

where

s = index of states, $s=1, \dots, t^n$

t = index of periods, $t=1, \dots, T-1$, where T is the last period under planning

i = index of options

$V_t^s(m)$ = value of the investment at the beginning of t , given state s is newly observed while option during $t-1$ was m

$C_t^s(i)$ = net cash flow at the beginning of t , given state s is newly observed while operating in option i

$E[V_{t+1}(i)]$ = expected value of $V_{t+1}(i)$ with respect to risk-neutral probabilities

r = risk-free rate of return in a period

$I(m \rightarrow i)$ = switching cost from option m to option i

F_t^s = set of feasible options at the beginning of t , given state s is newly observed

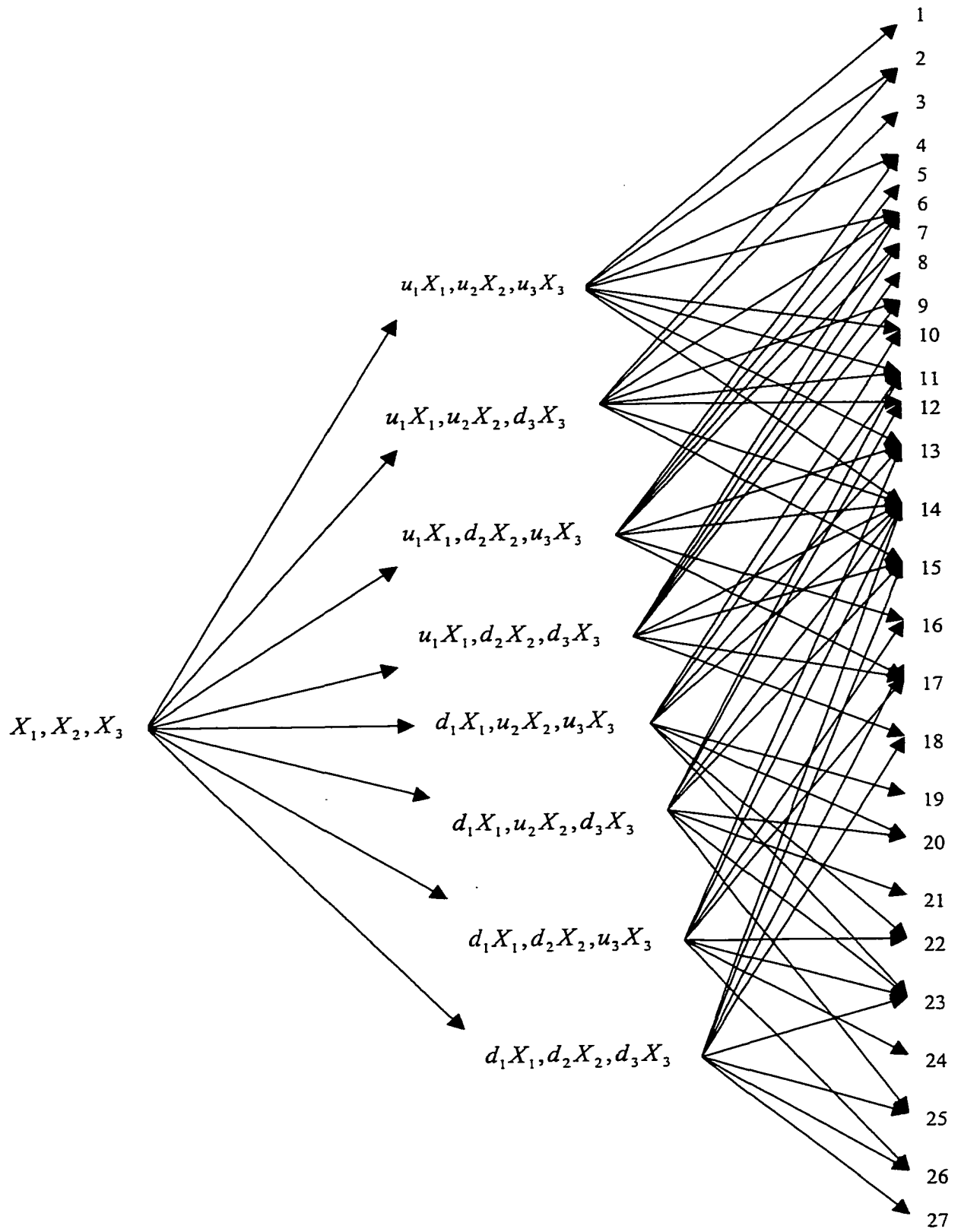


Figure 5.3 Evolution of three underlying assets

Table 5.1 Values of underlying assets in period 3

1	$u_1^2 X_1, u_2^2 X_2, u_3^2 X_3$	15	$u_1 d_1 X_1, u_2 d_2 X_2, d_3^2 X_3$
2	$u_1^2 X_1, u_2^2 X_2, u_3 d_3 X_3$	16	$u_1 d_1 X_1, d_2^2 X_2, u_3^2 X_3$
3	$u_1^2 X_1, u_2^2 X_2, d_3^2 X_3$	17	$u_1 d_1 X_1, d_2^2 X_2, u_3 d_3 X_3$
4	$u_1^2 X_1, u_2 d_2 X_2, u_3^2 X_3$	18	$u_1 d_1 X_1, d_2^2 X_2, d_3^2 X_3$
5	$u_1^2 X_1, d_2^2 X_2, u_3^2 X_3$	19	$d_1^2 X_1, u_2^2 X_2, u_3^2 X_3$
6	$u_1^2 X_1, u_2 d_2 X_2, u_3 d_3 X_3$	20	$d_1^2 X_1, u_2^2 X_2, u_3 d_3 X_3$
7	$u_1^2 X_1, u_2^2 X_2, u_3 d_3 X_3$	21	$d_1^2 X_1, u_2^2 X_2, d_3^2 X_3$
8	$u_1^2 X_1, d_2^2 X_2, d_3^2 X_3$	22	$d_1^2 X_1, u_2 d_2 X_2, u_3^2 X_3$
9	$u_1^2 X_1, u_2 d_2 X_2, d_3^2 X_3$	23	$d_1^2 X_1, u_2 d_2 X_2, u_3 d_3 X_3$
10	$u_1 d_1 X_1, u_2^2 X_2, u_3^2 X_3$	24	$d_1^2 X_1, d_2^2 X_2, u_3^2 X_3$
11	$u_1 d_1 X_1, u_2^2 X_2, u_3 d_3 X_3$	25	$d_1^2 X_1, u_2 d_2 X_2, d_3^2 X_3$
12	$u_1 d_1 X_1, u_2^2 X_2, d_3^2 X_3$	26	$d_1^2 X_1, d_2^2 X_2, u_3 d_3 X_3$
13	$u_1 d_1 X_1, u_2 d_2 X_2, u_3^2 X_3$	27	$d_1^2 X_1, d_2^2 X_2, d_3^2 X_3$
14	$u_1 d_1 X_1, u_2 d_2 X_2, u_3 d_3 X_3$		

If there are three projects (three underlying assets), the probability distribution for

$V_{t+1}(i)$ is as follows:

$$V_{t+1}(i) = \left[\begin{array}{l} V_{t+1}^{u_1 u_2 u_3}(i) \text{ with probability } p_{u_1} p_{u_2} p_{u_3} + \frac{\rho_{12} + \rho_{13} + \rho_{23}}{8} \\ V_{t+1}^{u_1 u_2 d_3}(i) \text{ with probability } p_{u_1} p_{u_2} p_{d_3} + \frac{\rho_{12} - \rho_{13} - \rho_{23}}{8} \\ V_{t+1}^{u_1 d_2 u_3}(i) \text{ with probability } p_{u_1} p_{d_2} p_{u_3} + \frac{-\rho_{12} + \rho_{13} - \rho_{23}}{8} \\ V_{t+1}^{u_1 d_2 d_3}(i) \text{ with probability } p_{u_1} p_{d_2} p_{d_3} + \frac{-\rho_{12} - \rho_{13} + \rho_{23}}{8} \\ V_{t+1}^{d_1 u_2 u_3}(i) \text{ with probability } p_{d_1} p_{u_2} p_{u_3} + \frac{-\rho_{12} - \rho_{13} + \rho_{23}}{8} \\ V_{t+1}^{d_1 u_2 d_3}(i) \text{ with probability } p_{d_1} p_{u_2} p_{d_3} + \frac{-\rho_{12} + \rho_{13} - \rho_{23}}{8} \\ V_{t+1}^{d_1 d_2 u_3}(i) \text{ with probability } p_{d_1} p_{d_2} p_{u_3} + \frac{\rho_{12} - \rho_{13} - \rho_{23}}{8} \\ V_{t+1}^{d_1 d_2 d_3}(i) \text{ with probability } p_{d_1} p_{d_2} p_{d_3} + \frac{\rho_{12} + \rho_{13} + \rho_{23}}{8} \end{array} \right.$$

We note that the above probabilities are risk-neutral probabilities. Based on these probabilities, we can calculate the expected value $E[V_{t+1}(i)]$. This will be discounted at the risk-free rate of return as in equation (5.1).

Switching from an option to another option requires a switching cost. If a switch is between the same option, the switching cost is assumed to be zero. Some switches are not possible. For example, switching from operating at a constructed capacity level to deferring the construction for the same project is not possible. Thus, the switching cost is assumed to be infinite.

We note that each option, given a state and a period, should be checked for feasibility. That is, there may be various constraints (such as financial or demand constraints) that make it impossible to exercise an option. For example, a utility may not have enough budget to construct three gas turbine power plants in a period. With the considerations of switching costs and constraints, the number of options that should be evaluated through backward dynamic programming will be greatly reduced.

The value of the investment at the beginning of the last period can be obtained as follows:

$$V_T^s(m) = \max_i (C_T^s(i) - I(m \rightarrow i))$$

s.t.

$$I(m \rightarrow i) = 0 \quad \text{for } i = m$$

$$i \in F_T^s$$

5.7 A Numerical Example for Three Correlated Projects

We now provide a numerical example with hypothetical data to illustrate our multiple-period lattice model. In this example, there are three correlated projects. Project 1 represents a potential gas turbine power plant, called power plant 1. Project 2 represents another potential gas turbine power plant, called power plant 2. Project 3 represents a potential wind power plant, called power plant 3. An eight-branch lattice is employed to represent the evolution of the three correlated underlying assets. Each project has the options to construct the power plant, to defer the construction of the power plant, to operate at the constructed capacity level, and to sell the constructed power plant at its salvage value. Therefore, the total number of options in a period is 64.

The planning horizon is 3 periods and the length of a period is 6 months. For simplicity, we assume that the construction of a power plant can be completed in a period. This assumption can be easily relaxed by dividing construction of a power plant into multiple stages and each stage can be completed in a period. The completion of a stage will provide the option to complete the following stage. Risk-free rate of return in a period, r , is assumed to be 0.05. The correlation coefficient between underlying assets 1 and 2, ρ_{12} , is assumed to be 0.5. The correlation coefficient between underlying assets 1 and 3, ρ_{13} , is assumed to be -0.6 . The correlation coefficient between underlying assets 2 and 3, ρ_{23} , is assumed to be -0.7 . The data for each potential power plant is listed in Table 5.2.

Table 5.2 Potential power plants

	Gas turbine 1 (power plant 1)	Gas turbine 2 (power plant 2)	Wind power (power plant 3)
Constructed capacity (MW)	100	150	50
Annual rate of return, μ_i	0.3	0.3	0.2
Annual standard deviation, σ_i	0.4	0.4	0.2
Construction cost (\$ million)	50	80	20
Market value of the underlying asset at the beginning of period 1	60	90	25

There exist two sets of constraints. The details of these constraints are listed in Table 5.3. We note that the total construction costs spent in a state cannot be greater than the budget restriction in that period. Also, we assume that the upper management has strategic requirements on capacity that is external to the real options model (i.e., the capacity requirements are parameters). The total capacity operated in a state cannot be less than the capacity restriction under that state.

With the input data, we first calculate the risk-neutral probabilities of the binomial lattices for each underlying asset as $p_{u_1} = 0.4735$, $p_{u_2} = 0.4735$, $p_{u_3} = 0.553$. The multiple

Table 5.3 Budget and capacity constraints

	$t=1$	$t=2$	$t=3$
Budget (US\$ million)	150 for all states	60 for all states	None for all states
Capacity requirement (MW)	None for all states	250 for first 6 states 230 for last 2 states	280 for first 26 states 220 for last state

for an up-move for underlying asset 1, 2, 3 is calculated as 1.327, 1.325, and 1.152, respectively. The multiple for a down-move for underlying asset 1, 2, 3 is calculated as 0.754, 0.754, and 0.868, respectively. Then, with the values of the correlation coefficients, we can calculate the risk-neutral probabilities for the eight branches emanating from each state in the eight-branch lattice as shown in Table 5.4.

Table 5.4 Probability for each branch

Branch	1	2	3	4	5	6	7	8
Probability	0.024	0.325	0.088	0.036	0.063	0.061	0.378	0.024

With the probabilities for the branches, the multiples for the underlying assets, and the initial values for the underlying assets, the lattice representing the evolution of the underlying assets is finally constructed.

Each option is associated with a three-period eight-branch lattice as the underlying assets. The net cash flows representing the payoffs of an option under different states (i.e., nodes in a lattice) can be estimated by using the values of the underlying assets. In this example, the payoff of the option to sell a power plant is assumed to be the value of its underlying asset. The payoff of the option to construct a power plant is assumed to be the negative value of its construction cost, which is assumed to be constant over time. The payoff of the option to operate a power plant in a period is assumed to be one-fourth of the value of its underlying asset in that period. Finally, the payoff of the option to defer the construction of a power plant is assumed to be zero.

We note that, in our three-period example, period 3 is not the last period of the life of the power plants. Instead, it represents the last period of a utility's planning. Thus, the

payoff in period 3 represents the cumulated payoff for all later periods. For example, the payoff for the option to operate at its constructed capacity level is assumed to be equal to 2 times the value of the underlying asset in period 3 if there are two up moves for the underlying asset in the previous two periods.

For some options, the net cash flows are set at negative infinity in some periods. For example, if the construction of a generation planning project requires one period to complete, then for that project the earliest feasible net cash flows for the option to operate at the constructed capacity occur in period 2. Therefore, the net cash flows are set at negative infinity in period 1.

5.7.1 Solution

In the first 26 states at $t=3$, the only option that satisfies the capacity constraints is the option to operate all three power plants. In the last state at $t=3$, four options satisfy the capacity constraint. They are the option to operate power plants 1, 2 and construct power plant 3, the option to operate power plants 1, 2 and defer the construction of power plant 3, the option to operate power plants 1, 2, 3, and the option to operate power plants 1, 2 and sell power plant 3. In all states at $t=2$, the above four options are the only options that satisfy both the capacity constraints and the budget constraints. At $t=1$, all 64 options satisfy the budget constraint.

After we examine constraints and obtain feasible options for all states, we examine the switching costs. There are four feasible options at $t=2$. However, only the option to operate all three power plants and the option to operate power plants 1, 2 and construct power plant 3 do not incur infinite switching costs to the option to operate all three power plants in the first 26 states at $t=3$. Since we already operate or construct power plant 3 at $t=2$,

it is impossible to switch to the option that involves the construction and the deferral of power plant 3 in the last state at $t=3$. Therefore, the options left in the last state at $t=3$ are the option to operate power plants 1, 2, 3 and the option to operate power plants 1, 2 and sell power plant 3. Then, at $t=1$, only the option to construct all three power plants and the option to construct power plants 1, 2 and defer the construction of power plant 3 do not incur infinite switching costs. By considering switching costs, we reduce the originally 64 feasible options to less than or equal to 2 options in all states that need to be evaluated through backward dynamic programming.

By backward dynamic programming, the optimal options are to construct all three power plants at $t=1$, operate all three power plants in all states at $t=2$, operate all three power plants in the first 26 states at $t=3$, and operate power plants 1, 2 and sell power plant 3 in the last state at $t=3$. The corresponding optimal value of the investment at the beginning of $t=1$ is \$ 156.96 millions.

Through this numerical example, we proposed a two-step procedure to reduce the number of options that need to be evaluated by backward dynamic programming. First, we eliminated the options that violate the constraints. Then, starting from the last period, we eliminated the options that induce infinite switching costs. Via this procedure, the size of the dynamic programming problem can be greatly reduced.

5.8 Concluding Remarks

In this chapter, we first presented an eight-branch lattice model for three correlated projects. From this model, we constructed a lattice model for multiple correlated projects ($n \geq 2$). Then, we compared our lattice model with other lattice models. Next, we

investigated the roles of correlation coefficients for multiple correlated projects. Finally, an illustrative numerical example with three correlated projects was provided to show how sequential decisions can be made for a utility with multiple correlated projects.

We reviewed the several different ways to obtain the probabilities for branches of the lattice. It will be interesting to compare and contrast the advantages and disadvantages of each way quantitatively.

CHAPTER 6. GENERATION PLANNING WITH OUTAGE COSTS

6.1 Introduction

In this chapter, we present an extension of a traditional generation planning model that is in contrast to the real options models discussed thus far. In this model, we incorporate both forced utility outage cost and forced customer outage cost. By including the forced customer outage cost, we attempt to take the customer satisfaction level into consideration in so far as the generation planning is concerned.

The purpose of traditional generation planning models is to determine the generation units to be constructed and the amount of power to be produced while the total cost (fixed and production cost) to a utility is minimized. We note that the traditional models only focus on relatively easy-to-quantify factors such as fixed and production cost. Forced customer outage cost and forced utility outage cost are not considered in most of the generation planning literature.

In this chapter, we develop a generation planning model, explicitly considering forced customer outage cost and forced utility outage cost. Since utility outage cost is the revenue loss to a utility, which can be obtained from published data, this chapter focuses on how to obtain customer outage cost. To obtain customer outage cost, we first review the quantification approaches for customer outage cost in the literature. Because the diversity of the quantification approaches presents some difficulties for utilities to make generation planning decisions (e.g., consistency of assumptions), we suggest a single quantification approach that can be used in generation planning. This single approach provides us the customer outage cost per unsupplied MWh. Then, we show how the expected amount of

outage can be obtained via a linear regression model. By multiplying the customer outage cost per unsupplied MWh by the expected amount of outage, the expected customer outage cost can be obtained. Finally, we develop a mixed integer linear programming model incorporating customer outage cost and utility outage cost for generation planning, and illustrate the key features of the model via a numerical example.

The organization of this chapter is as follows. In Section 6.2, we review the quantification approaches for customer outage cost in the literature. In Section 6.3, we introduce a linear regression model to estimate the expected amount of forced outage. In Section 6.4, we present how the expected outage cost can be obtained using the results of Sections 6.2 and 6.3. In Section 6.5, a mixed integer linear programming model with the expected outage cost is formulated. Section 6.6 provides a comprehensive numerical example to illustrate the applicability of the model. Finally, concluding remarks are provided in Section 6.7.

6.2 Quantification Approaches for Customer Outage Costs

The quantification approaches for customer outage costs in the literature can be classified into three groups: (1) proxy approaches, (2) sophisticated econometric approaches, and (3) survey approaches. This classification depends on the nature of how customer outage costs are estimated. Proxy approaches apply secondary data (e.g., backup power costs, wage rates) to estimate customer outage costs. Sophisticated econometric approaches investigate the possible loss to firms in case of outage. For example, the possible outage costs to a firm could include damage to material, forgone profit, and wage payment to idle workers if an outage occurs. On the other hand, survey approaches inquire customers to identify their

possible loss in different hypothetical scenarios. Finally, we observe that the study of outage costs has often focused on only a specific customer type such as residential, industrial, and commercial types. We now elaborate on these specific customer types.

(1) Residential type

Several papers have argued that the loss of leisure is the main element of outage costs to residential customers. Munasinghe [38] claims that the enjoyment of leisure is usually restricted in the evening. Leisure activities such as television-watching and reading require electricity. Therefore, there will be very few substitution possibilities for those electricity-dependent activities if an outage occurs. The same paper also develops a model of estimating the outage costs to the residential customer type by using a utility function (i.e., a proxy approach of wage rate). The key idea is that an outage interrupts the preferred pattern of consumption and thus leads to a loss. The author claims that the wage rate could be a good proxy for residential outage costs.

(2) Industrial type

Bental and Ravid [9] claim, "industrial customers would purchase the backup generating power if the expected gain from the marginal self-generated kWh equals the expected loss from the marginal kWh that is not supplied by the utility. Hence, the expected marginal cost of backup power may serve as an estimate for the marginal outage costs" (i.e., a proxy approach). The sophisticated econometric approach is another common approach in the literature. Tishler [50] develops a model to measure expected outage costs by

using a production function. In that paper, the costs of outage are contributed by various sources such as a possible reduction in productivity.

(3) Commercial type

Sanghvi [42] finds that some problems may arise in developing an appropriate approach to estimate the outage costs for commercial customers because the definition of commercial activities is often unclear. Sanghvi claims that "large apartment buildings, small pizza parlors, and moderate size manufacturing firms can be classified as the commercial user types."

However, a large part of commercial activities are production activities. The outage costs of those activities can be estimated by using the same approaches as we do for the industrial customer type. Moreover, some commercial activities, such as large shopping centers, can be viewed as having characteristics similar to the residential customer type and can be analyzed by using the residential approaches.

6.2.1 Difficulties in Reconciling Different Approaches

As presented in the previous section, the study of outage costs has often focused on only a specific customer type. If different approaches are employed for different customer types, and if the resulting outage costs are aggregated, this could present a serious problem to the decision maker of the generation planning. For example, let us suppose that Tishler's model [50] (a sophisticated econometric approach) is applied to estimate the outage costs of industrial customers. At the same time, let us suppose that Munasinghe's model [38] (a proxy approach) based on the wage rate is applied to estimate the outage costs of residential customers. Finally, let us suppose that a survey approach is applied to estimate the outage

costs of commercial customers. Aggregating the outage costs over all customers in this case will be difficult to justify because (1) underlying assumptions are contradictory. For example, Tishler's model assumes that a key component of the outage cost is the foregone profit of a firm. On the other hand, Munasinghe's model assumes that a key component of the outage cost is the wage (i.e., revenue) of a resident. Hence, if these outage costs are aggregated, then such a number reflects neither the profit component nor the revenue component, making any interpretation inaccurate and misleading. Also, (2) resulting outcomes have their own unique tendencies (e.g., survey approaches tend to overestimate; see e.g., Sanghvi [42]). Therefore, aggregating outage costs from different approaches makes it difficult for utilities to evaluate generation planning projects analytically and objectively.

*A few papers have demonstrated that we should be careful when we compare outage costs. Tishler [50] says, "direct comparison of outage cost estimates from different studies should be viewed with caution." Moreover, Sanghvi [42] states, "there exists the factors that sometimes limit the ability to strictly compare different outage cost estimates. Often the estimates are based upon different methodologies, assumptions, economic and demographic mixes and local conditions, and for different outage descriptors."

6.2.2 A Single Approach for All Customers: Proxy Approach

Caves et al. [14] note that surveys are the principal source of information on customer outage costs. Survey approaches inquire customers to identify their possible response in different hypothetical scenarios. The customers would be inquired how much it would cost them to adjust to this power outage. However, since most consumers do not have much

knowledge and quantification experience, it is almost impossible for them to properly estimate the costs under the hypothetical scenarios.

Sophisticated econometric approaches have been commonly used for estimating industrial and commercial outage costs. But there will be some difficulties if we apply them for residential customers. Caves et al. [14] claim that residential customer type could be viewed as industrial customer type if we consider household activities as a production process where electricity is an input and leisure activities are outputs. Then residential outage cost is the market value of those household activities lost due to an outage. It is, however, very difficult to obtain a complete listing of household activities and their market values.

In contrast to the survey approach and the sophisticated econometric approach, the proxy approach is conceptually simple and the data for proxies are often easily obtainable. Even though the proxy approach may contain theoretical deficiencies and/or inaccurate estimates (see e.g., Woo and Pupp [59]), relative to the sophisticated econometric approach, for a quick and first-order approximation, the proxy approach can be effective and practical. Furthermore, as the data accumulate, the accuracy of this approach can improve. Hence, in this chapter, we choose to employ the proxy approach as the single quantification approach for all customer types.

To illustrate how the proxy approach is applied to quantification of outage costs of all customer types, we will employ the wage rate as the proxy for the outage cost of residential customers and the backup power cost as the proxy for the outage cost of industrial and commercial customers.

6.2.3 Wage Rate Method for Residential Customers

Munasinghe [38] demonstrates that the wage rate can be a good estimate for residential outage costs. Specifically, he develops a linear regression model as follows:

$$OC_i = b_1 + b_2 Y_i + e_i \quad (6.1)$$

where i , OC_i , e_i , and Y_i are the index of customer, the outage cost per hour for customer i , the random disturbance term for customer i , and the net income earning per hour for customer i .

A partial collection of data provided in Munasinghe [38] is shown in Table 6.1. We note that the Brazilian currency is used.

Given this partial collection of data, we estimate the coefficients to be $b_1 = 3.097$ and $b_2 = 0.922$, yielding a good fit with $R^2 = 0.806$. Hence, the proxy relation between the outage cost and the wage rate for customer i is $OC_i(\text{Brazil}) = 3.097 + 0.922 Y_i$. In U.S. currency, this

Table 6.1 Residential consumer survey results (Cr\$ per hour)

Net Income Earning Rate	Outage Cost	Net Income Earning Rate	Outage Cost
10.5	12	27.7	20
11.1	15	30.8	35
14.2	10	40.3	30
15.4	20	40.3	30
18.5	25	45.5	50
18.5	35	48.0	60
19.7	20	55.1	45
21.5	15	64.6	55
21.5	25	64.6	60
24.6	15	73.9	85
24.6	40		

* U.S. \$1 = Cr \$12.35 (end 1976)

proxy relation is converted as $OC_i(\text{U.S.}) = 0.251 + 0.922Y_i$ (note that this converted model is only for illustrative purpose).

We note that OC_i is the outage cost per hour for customer i . To incorporate residential customer outage costs in our generation planning model, we essentially need outage costs per kWh. In what follows, we show via a numerical example how the outage cost per kWh can be obtained from the outage cost per hour for customer i , OC_i .

Step 1. Suppose that a utility has obtained the following data for customer i :

The wage rate Y_i : US\$11.12 per hour. i.e., the corresponding outage cost OC_i is US\$10.5 per hour (because $OC_i = 0.251 + 0.922Y_i$).

The total number of leisure hours: 1000 hours per year.

The total electricity consumption during leisure periods: 700 kWh.

Step 2. The average power consumption in leisure activities for customer i can be calculated as

$$\frac{700 \text{ kWh}}{1000 \text{ h}} = 0.7 \text{ kW}$$

Step 3. Outage cost per kWh for customer i can be calculated as

$$\frac{\text{US\$10.5 per hour}}{0.7 \text{ kW}} = \text{US\$15 per kWh} = \text{US\$15000 per MWh}$$

6.2.4 Backup Power Cost Method for Industrial and Commercial Customers

Thus far we have shown how the outage cost is estimated for the residential customers via the wage rate method. Now we will show how the outage cost is estimated for the industrial and commercial customers via the backup power cost method. This method is similar to the work by Bental and Ravid [9].

The backup power cost method assumes that customers act rationally and would like to insure themselves against the damage caused by electricity outages. Because insurances are unavailable, industrial and commercial customers will acquire backup generators. Therefore, the costs of self-generating a kWh of the unsupplied utility electricity may serve as an estimate for the expected avoided outage costs due to this self-generating kWh.

The costs of generating backup power consist of two items: the capacity cost of constructing the backup facility, and operating cost. So the average outage costs per kWh can be expressed as

$$AC = \frac{b}{H} + v \quad (6.2)$$

where b , H , and v are the annual capacity cost of backup facility per kW, the expected duration of outage per year, and the operating cost of generating power per kWh, respectively. Let us assume, in this section, that the interest rate is 7% and the capacity cost of backup facility per kW is \$150. Also, we employ straight-line depreciation over ten years, and the tax rate is 20%. Then the annual capacity cost of backup facility per kW, b , can be calculated as

$$b = 150(A/P)_{10}^{7\%} - \frac{150}{10} \frac{20}{100} = 18.357$$

where (A/P) is the uniform series worth of a present sum and the value of $(A/P)_{10}^{7\%}$ is found as 0.14238 in engineering economy textbooks. Other key parameter values are as follows: The expected duration of outage per year, H , is taken to be eight hours per year. Operating cost per kWh, v , is \$0.5. Using the above equation, the average outage cost per

Table 6.2 Average outage cost

<i>H</i>	8	16	24	48	96
<i>AC</i>	2.79	1.65	1.26	0.88	0.69

kWh, *AC*, is \$2.79. Table 6.2 below shows the expected durations of outage per year and the corresponding average outage cost.

6.3 Expected Amount of Outage

The proxy approach provides us a simple and quick tool to estimate customer outage costs per unsupplied kWh. To incorporate outage costs into a capital budgeting model for generation planning, the expected amount of outage in MWh is needed. Hence, first, we show how the expected amount of outage in MW can be estimated via a linear regression model.

Due to the random nature of outage, Ammons [1], Irisari [24], Ghajar and Billinton [19], Niimura and Kermanshahi [39], and Bloom [11] have demonstrated that the expected amount of outage can be computed from the available capacity and demand distributions. They have presented that the levels of customer demand and available capacity will determine the expected amount of outage. From this basis, we construct a linear regression model consisting of the expected amount of outage as the dependent variable and the levels of customer demand and available capacity of a utility as the independent variables. The linear regression model is given by

$$FO_{it} = \alpha \sum_{j=1}^J a_{jt} U_{jt} y_j + \beta P_{it} + \gamma \quad (6.3)$$

where

- j = the index of either an existing generation unit or a generation expansion planning project. $j = 1, \dots, \bar{j}$ are for existing generation units, $j = \bar{j} + 1, \dots, J$ are for generation expansion planning projects
- i = the index of intervals in a period, e.g., peak/off-peak durations, $i = 1, \dots, I$
- t = the index of period of time, e.g., year, $t = 1, \dots, T$
- FO_{it} = the expected amount of outage at a point in time in interval i of period t (MW) cumulative of all customer types
- a_{jt} = the availability factor for generation unit j in period t , $0 \leq a_{jt} \leq 1$
- U_{jt} = the upper bound on production level (capacity) on generation unit j in period t (MW)
- y_j = the binary variable indicating whether generation expansion planning project j is to be constructed. For $j = 1, \dots, \bar{j}$, $y_j = 1$. For $j = \bar{j} + 1, \dots, J$, $y_j = 1$ implies construction, $y_j = 0$ implies no construction
- P_{it} = the average customer demand at a point in time in interval i of period t (MW)
- α, β, γ = the coefficients needing to be estimated

The expected amount of outage to a specific customer type can be obtained by multiplying the expected amount of outage by the fraction of the total energy consumption consumed by that specific customer type (see e.g., Lo et al. [31]). Hence, the expected amount of outage in MWh for customer type w in interval i of period t is

$$k_{wt}FO_{it}h_{it} \quad (6.4)$$

where

w = the index of customer type (1: residential type, 2: commercial type, 3: industrial type)

k_{wt} = the fraction of the total energy consumption consumed by customer type w in period t ($\sum_{w=1}^3 k_{wt} = 1$)

h_{it} = the number of hours in interval i of period t

6.4 Expected Outage Cost

We note that the expected outage cost in our model consists of expected utility outage cost and expected customer outage cost. Expected utility outage cost is the expected revenue loss to a utility. The expected revenue loss to a utility can be obtained by multiplying the expected amount of outage in MWh by revenue per MWh. Expected customer outage cost can be obtained by multiplying the expected amount of outage in MWh by customer outage cost per unsupplied MWh. Hence, the expected discounted outage cost from customer type w in interval i of period t is

$$(C_{wit}^U + C_{wit}^C)k_{wt}FO_{it}h_{it} \quad (6.5)$$

where

C_{wit}^U = the discounted revenue per MWh to a utility from a representative customer type w in interval i of period t

C_{wit}^C = the discounted outage cost per unsupplied MWh for a representative customer of customer type w in interval i of period t . (i.e., C_{it}^C can be

estimated by the wage rate method, C_{2it}^C and C_{3it}^C can be estimated by the backup power cost method)

Hence, the total expected discounted outage cost for all customer types over the entire planning horizon is

$$\sum_{t=1}^T \sum_{i=1}^I \sum_{w=1}^3 (C_{wit}^U + C_{wit}^C) k_{wt} FO_{it} h_{it} \quad (6.6)$$

6.5 The General Model

The objective of this model is to minimize the total cost consisting of discounted fixed, production, utility outage, customer outage, and outside purchase costs minus the discounted revenue from selling any power outside. This objective is subject to the constraints of production, purchase, sale, demand, budget, and reliability. The decision variables representing generation expansion planning projects are binary (0-1) for the project selection purpose. The decision variables representing the amount of power produced, the portion of construction budget not used, the amount of outside purchase, and the amount of outside sale are all continuous variables. Hence, the resulting mixed integer linear programming model is formulated as follows:

Minimize

$$y_j, x_{jit}, S_t, OP_{it}, OS_{it}$$

$$\begin{aligned} & \sum_{j=1}^J f_j y_j + \sum_{t=1}^T \sum_{i=1}^I \sum_{j=1}^J F_{jit} x_{jit} h_{it} + \sum_{t=1}^T \sum_{i=1}^I \sum_{w=1}^3 (C_{wit}^U + C_{wit}^C) k_{wt} FO_{it} h_{it} \\ & + \sum_{t=1}^T \sum_{i=1}^I PC_{it} OP_{it} h_{it} - \sum_{t=1}^T \sum_{i=1}^I SP_{it} OS_{it} h_{it} \end{aligned} \quad (6.7)$$

Subject to

$$L_{jt}y_j \leq x_{jit} \leq a_{jt}U_{jt}y_j \quad (6.8)$$

$$0 \leq OP_{it} \leq OP_{it-\max} \quad (6.9)$$

$$0 \leq OS_{it} \leq OS_{it-\max} \quad (6.10)$$

$$OS_{it} \leq \sum_{j=1}^J x_{jit} + OP_{it} - P_{it} \quad (6.11)$$

$$S_t - (1 + r_{t-1})S_{t-1} + \sum_{j=1}^J C_{jt}y_j = B_t \quad (6.12)$$

$$\sum_{j=1}^J a_{jt}U_{jt}y_j \geq (1 + m)P_t \quad (6.13)$$

$$\sum_{j \in \{J_k\}} y_j \leq 1 \quad (6.14)$$

$$y_j = 1 \quad \text{for } j = 1, \dots, \bar{j}$$

$$y_j \in \{0,1\} \quad \text{for } j = \bar{j} + 1, \dots, J \quad (6.15)$$

$$S_t \geq 0, x_{jit} \geq 0, OP_{it} \geq 0, OS_{it} \geq 0 \quad (6.16)$$

$$j = 1, \dots, J$$

$$i = 1, \dots, I$$

$$t = 1, \dots, T$$

$$k = 1, \dots, K$$

where

B_t = the construction budget available in period t

C_{jt} = the construction expense required for constructing generation expansion
planning project j in period t

- f_j = the sum of discounted fixed costs (e.g., taxes, insurance, maintenance, etc.) associated with generation unit j over the planning horizon
- F_{jit} = the discounted production cost (i.e., variable cost and not fixed cost) per unit of energy output associated with generation unit j in interval i of period t (\$/MWh)
- L_{jt} = the lower bound on production level on generation unit j in period t (MW)
- m = the reserve margin for extra demand above P_t
- $OP_{it-\max}$ = the maximum amount of power can be purchased in interval i of period t (MW)
- $OS_{it-\max}$ = the maximum amount of power can be sold in interval i of period t (MW)
- PC_{it} = the discounted outside purchase cost in interval i of period t (\$/MWh)
- r_t = the rate of return on short term investments for period t , $r_0 = 0$
- OP_{it} = the amount of outside purchase in interval i of period t (MW)
- OS_{it} = the amount of outside sale in interval i of period t (MW)
- S_t = the construction budget unused in period t , $S_0 = 0$
- SP_{it} = the discounted selling price per unit of energy output in interval i of period t (\$/MWh)
- $\{J_k\}$ = the set of the indices of the generation expansion planning projects representing all possible beginning years of the same potential generation unit k ($k = 1, \dots, K$)
- x_{jit} = the power produced by generation unit j in interval i of period t (MW)

P_t = the peak demand during period t (MW)

In this formulation, we consider both existing generation units and generation expansion planning projects. We note that a generation expansion planning project represents a particular type of generation unit to be constructed at a given time with a given capacity. If the value of a binary decision variable is one, the corresponding project is selected for construction. If the value of a binary decision variable is zero, the corresponding project is not selected for construction. On the other hand, we note that the values of binary decision variables corresponding to the existing generation units are fixed at one.

The objective function is the sum of discounted fixed cost ($\sum_{j=1}^J f_j y_j$), discounted production cost ($\sum_{t=1}^T \sum_{i=1}^I \sum_{j=1}^J F_{jit} x_{jit} h_{it}$), discounted utility and customer outage cost ($\sum_{t=1}^T \sum_{i=1}^I \sum_{w=1}^3 (C_{wit}^U + C_{wit}^C) k_{wt} FO_{it} h_{it}$), and discounted outside purchase cost ($\sum_{t=1}^T \sum_{i=1}^I PC_{it} OP_{it} h_{it}$) minus discounted revenue from selling any power outside ($\sum_{t=1}^T \sum_{i=1}^I SP_{it} OS_{it} h_{it}$). We note that the objective function is measured in present worth because the planning horizon can be expensive (ten years or more). We further note that, for computational tractability, we choose a linear production cost function (cf., nonlinear production cost function; see e.g., Ammons and McGinnis [2]). The linear production cost function can be found in numerous papers (see e.g., Anderson [4], Hobbs [20], and Hobbs and Centolella [21]).

Constraints (6.8) state that the amount of power to be produced is bounded from above by the available capacity and bounded from below by the minimum required production level for each generation unit. We note that these constraints are non-trivial only

if $y_j = 1$. Furthermore, we note that when a generation expansion planning project is under construction, the corresponding L_{jt} and $a_{jt}U_{jt}$ will be set at zero. This implies that no power will be produced during the construction period.

Constraints (6.9) and (6.10) state that the amount of outside purchase and the amount of outside sale cannot exceed some given upper limits. These limits may be determined by the contracts between the utility we are modelling and other utilities. In Zhang et al. [60], Christoforidis et al. [16], and Bart et al. [6], an upper limit for each purchase or sale is considered. We note that, for simplicity, in this chapter we do not distinguish each purchase or sale. Instead, we have the upper limits for the amount of outside purchase and the amount of outside sale for an entire time interval.

Constraints (6.11) state that the sum of power produced plus the amount of outside purchase minus the amount of outside sale is greater than or equal to the expected customer demand. As the electric power industry becomes more competitive, there will be an increasing number of opportunities for outside purchase and outside sale. These constraints capture the utilities' attempt to satisfy customer demand via self-producing and outside purchase. Also, these constraints consider the opportunities of selling extra power to other utilities.

Constraints (6.12) state that in each period (i.e., year) a certain amount of budget is available for the construction of generation expansion planning projects. The construction budget that is left in the current period will be carried forward to the next period for construction. The construction budget carried forward earns interest at some specified rate of return on short term investment. We note that B_t , C_{jt} , and S_t are not measured in present

worth. We also note that, for generation expansion planning projects, the corresponding C_{jt} 's will be set at zero before the start of construction and after the end of construction. For existing generation units, all C_{jt} 's are set at zero because no construction expense is required.

Constraints (6.13) state that the sum of available capacity is greater than or equal to the sum of peak demand and reserve margin. The parameter P_t is the peak demand in period t and can be viewed as the maximum amount of demand among all intervals in a same period. We note that both outage cost and reserve margin (for reliability purpose) are considered in our model. Due to the random nature of outage, the improvements in reliability can help utilities reduce the expected amount of outage but cannot completely eliminate it. Hence, we incorporate the outage cost in the objective function and have reserve margin as reliability constraints in our model.

For a potential generation unit k ($k = 1, \dots, K$), there may be a multiple number of possible beginning years of construction. Each possible beginning year of construction represents a generation expansion planning project. Hence, a potential generation unit k may have a multiple number of generation expansion planning projects. The set J_k in constraints (6.14) consists of the indices of the generation expansion planning projects for the same potential generation unit k . We assume that once a project begins, the project will continue to completion. Constraints (6.14) state that at most one generation expansion planning project (one beginning year) can be selected for each potential generation unit.

Constraints (6.15) state that there are \bar{j} existing generation units and $J - \bar{j}$ generation expansion planning projects. Finally, constraints (6.16) state that the continuous decision variables are nonnegative.

6.6 Numerical Example

In this section, we illustrate some key features of the model via a hypothetical numerical example. In this numerical example, we have a two-period planning horizon ($T = 2$) with only one interval i (i.e., the interval is one year). Therefore, the subscript i is ignored in the notations of this example. There are two existing generation units and two potential generation units ($\bar{j} = 2, K = 2$). We assume that all potential generation units can be constructed at any period during the planning horizon. Hence, there are four generation expansion planning projects (i.e., $J = 6$, the first two are existing generation units, the last four are generation expansion planning projects). Utilizing the notation J_k defined in previous section, we note that: for $k = 1$, $J_1 = \{3,4\}$, for $k = 2$, $J_2 = \{5,6\}$. This implies that the potential generation unit 1 is associated with projects 3 and 4. Similarly, the potential generation unit 2 is associated with projects 5 and 6. We note that this two-period example can be extended to a large-scale planning problem which may span many periods. We note that a period may be much longer than a year. This period represents a time framework at which a critical generation expansion planning decision can be made.

For simplicity, in this model, we assume that in any period a project can be selected for construction and the power will be available in the same period (i.e., the duration of a period may be much longer than a year). We note, however, it is possible to elaborately

model the construction aspect by splitting the period into a multiple number of years, and extend the model accordingly (e.g., if a project is under construction for a particular year, then there will be no production during the year.).

6.6.1 Input Data

First, we need to estimate the coefficients of the linear regression model for the expected amount of outage. For this, we employ the data in Wang and Min [54] to produce Table 6.3.

Table 6.3 Outage data

Expected amount of forced outage (MW)	Demand (MW)	Capacity (MW)
3.1585	126	200
3.62	81	125
2.8324	89	100
3.33	100	150
1.2245	106	150
2.779	69	75
4.75	95	150
4.0875	115	150
0.39	13	25
1	20	50
6.175	115	150

Given Table 6.3, we can estimate the coefficients α , β , and γ of the linear regression model. By using the SAS software package (SAS Institute, Inc. [44]), we estimate the coefficients as $\alpha = -0.009502$, $\beta = 0.044012$, and $\gamma = 0.459070$.

The hypothetical input data (similar to those found in Wang and Min [54]) for existing generation units and generation expansion planning projects are shown in Tables 6.4

and 6.5, respectively. The reserve margin (m) is assumed to be 0.05. The periodic data are provided in Table 6.6.

In what follows, we assume the data is applicable to period 1 as well as period 2. We need data to estimate customer outage cost and utility outage cost. To estimate the outage

Table 6.4 Data for existing generation units

	Fixed cost (US\$)	Production cost(\$/MWh)	Capacity (MW)	Lower bound (MW)	Availability factor
Unit 1	$f_1 = 5,735,640$	$F_{11} = 17$	$U_{11} = 1,073$	$L_{11} = 50$	$a_{11} = 0.85$
		$F_{12} = 17$	$U_{12} = 1,073$	$L_{12} = 35$	$a_{12} = 0.90$
Unit 2	$f_2 = 2,500,000$	$F_{21} = 17$	$U_{21} = 598$	$L_{21} = 20$	$a_{21} = 0.76$
		$F_{22} = 17$	$U_{22} = 600$	$L_{22} = 30$	$a_{22} = 0.90$

Table 6.5 Data for generation expansion planning projects

	Fixed cost (US\$)	Production cost(\$/MWh)	Capacity (MW)	Lower bound (MW)	Availability factor	Construction cost (US\$)
Potential generation unit 1 constructed in period 1	$f_3 = 5,000,000$	$F_{31} = 17$	$U_{31} = 900$	$L_{31} = 45$	$a_{31} = 0.95$	$C_{31} = 10,000,000$
		$F_{32} = 17$	$U_{32} = 900$	$L_{32} = 45$	$a_{32} = 0.95$	$C_{32} = 0$
Potential generation unit 1 constructed in period 2	$f_4 = 2,500,000$	$F_{41} = 0$	$U_{41} = 0$	$L_{41} = 0$	$a_{41} = 0$	$C_{41} = 0$
		$F_{42} = 17$	$U_{42} = 900$	$L_{42} = 45$	$a_{42} = 0.95$	$C_{42} = 11,000,000$
Potential generation unit 2 constructed in period 1	$f_5 = 3,000,000$	$F_{51} = 18$	$U_{51} = 660$	$L_{51} = 35$	$a_{51} = 0.95$	$C_{51} = 15,000,000$
		$F_{52} = 18$	$U_{52} = 660$	$L_{52} = 35$	$a_{52} = 0.95$	$C_{52} = 0$
Potential generation unit 2 constructed in period 2	$f_6 = 1,500,000$	$F_{61} = 0$	$U_{61} = 0$	$L_{61} = 0$	$a_{61} = 0$	$C_{61} = 0$
		$F_{62} = 18$	$U_{62} = 660$	$L_{62} = 35$	$a_{62} = 0.95$	$C_{62} = 16,000,000$

Table 6.6 Periodic data

	h_t (hour)	PC_t (\$/MWh)	SP_t (\$/MWh)	$OP_{t-\max}$ (MW)	$OS_{t-\max}$ (MW)	P_t (MW)	r_t	B_t (US\$)
Period 1 ($t = 1$)	8,760	30	30	1,000	1,000	1,000	0.1	25,000,000
Period 2 ($t = 2$)	8,760	30	30	1,000	1,000	1,000	0.1	25,000,000

cost for the residential customers, we first assume that the wage rate for a representative residential customer is US\$11.12 (see e.g., U.S. Bureau of Labor Statistics [52]). Then, we apply the steps of the wage rate method and the data provided in the section of the wage rate method. The outage cost per MWh for a representative residential customer is calculated as US\$15000 ($C_{11}^C = 15000$). To estimate the outage cost for the commercial and industrial customers, we apply the backup power cost method. We assume that the expected duration of outage per year for a representative commercial and industrial customer is 16 hours (hypothetical). Then, from Table 6.2, the outage cost per MWh for a representative commercial and industrial customer is US\$2790 ($C_{21}^C = C_{31}^C = 2790$).

Since utility outage cost per unsupplied MWh is essentially the revenue per MWh to a utility, we need data for the revenue per MWh (price of electricity) from all customer types. Based on some government publications, the revenue per MWh to a utility from a representative residential, commercial, and industrial customer is US\$84, \$77, and \$47 (see e.g., U.S. Bureau of the Census [53]), respectively ($C_{11}^U = 84, C_{21}^U = 77, C_{31}^U = 47$). Finally, the fraction of the total energy consumption consumed by residential, commercial, and industrial customers are 31% (k_{11}), 19% (k_{21}), and 50% (k_{31}), respectively (see e.g., Lo et al. [32]).

6.6.2 Optimal Solution

We solve this example by using the LINDO software package (LINDO Systems Inc. [29]). The details of the optimal solution are as follows:

For the existing generation units 1 and 2, we have the binary variables $y_1 = y_2 = 1$. For the potential generation units 1 and 2, it is optimal to be constructed in period 1 (i.e., $y_3 = 1, y_4 = 0, y_5 = 1, y_6 = 0$). The value of the objective function is US\$1,998,480,429 (recall this figure is not for one year because each period will consist of multiple years).

As for the optimal solutions for the continuous variables for period 1, we have the following. The power produced by the existing generation units 1 and 2 is 655.52 MW and 454.48 MW, respectively ($x_{11} = 655.52, x_{21} = 454.48$). The power produced by the potential generation units 1 and 2 is 855 MW and 35 MW, respectively ($x_{31} = 855, x_{51} = 35$). The amount of outside purchase is 0 ($OP_1 = 0$). The amount of outside sale is 1,000 MW ($OS_1 = 1,000$). The construction budget unused is 0 ($S_1 = 0$).

As for the optimal solutions for the continuous variables for period 2, we have the following. The power produced by the existing generation units 1 and 2 is 570 MW and 540 MW, respectively ($x_{12} = 570, x_{22} = 540$). The power produced by the potential generation units 1 and 2 is 855 MW and 35 MW, respectively ($x_{32} = 855, x_{52} = 35$). The amount of outside purchase is 0 ($OP_2 = 0$). The amount of outside sale is 1,000 MW ($OS_2 = 1,000$). The construction budget unused is US\$25,000,000 ($S_2 = 25,000,000$).

We observe that both of the potential generation units are constructed in period 1. Since our model provides the opportunities for outside purchase and sale, the purchase cost and the selling price would influence a utility's decisions on the amount of power produced.

In our example, relative to the production price, the selling price is high. We believe that this high selling price makes a utility willing to produce more power than the customer demand, and sell the extra power to other utilities. Since it is profitable to sell in both periods 1 and 2, the optimal decision is to construct and operate from period 1 on.

6.7 Concluding Remarks

In this chapter we first examined the quantification approaches for customer outage cost in the literature. Next, we showed how the expected amount of outage can be obtained via a linear regression model. The multiplication of the outage cost per unsupplied MWh by the expected amount of outage provides the expected customer outage cost. Similarly, the multiplication of the revenue per MWh to a utility by the expected amount of outage provides the expected utility outage cost. Expected customer outage cost and expected utility outage cost are the components of expected outage cost. Finally, we developed a mixed integer linear programming model incorporating expected outage cost for generation planning. Via this model, the selection of generation expansion planning projects, the amount of power produced, the portion of construction budget not used, the amount of outside purchase, and the amount of outside sale can be determined.

For future research, it would be worthwhile to rigorously determine quantitative conditions under which the traditional generation planning is still recommendable instead of the generation planning models under uncertainties such as the models based on the real options approaches.

CHAPTER 7. COMPARISONS OF REAL OPTIONS APPROACH VS. TRADITIONAL APPROACH FOR GENERATION PLANNING

7.1 Introduction

In this chapter, we compare the multiple-project multiple-option model in Chapter 5 with the traditional generation planning model in Chapter 6 to illustrate the key features of the real options approach vs. the traditional approach for generation planning. Such an attempt is a first step toward understanding the two different approaches.

The multiple-project multiple-option model in Chapter 5 is useful when there are multiple projects and the projects are correlated. By correlation, we mean that the market values of the completed power plants are correlated. Via this model, we can determine the optimal sequential strategic decisions over options for correlated projects.

In the traditional generation planning model in Chapter 6, we incorporated both forced utility outage cost and forced customer outage cost. By including the forced customer outage cost, we attempted to take the customer satisfaction level into account. Via this model, we can determine the generation units to be constructed and the amount of power to be produced while the total cost to a utility is minimized.

This chapter is organized as follows. Section 7.2 provides generation planning process based on the real options approach and the traditional approach. Sections 7.3, 7.4, and 7.5 compare the multiple-project multiple-option model and the traditional model in terms of decision variables, objective function, and constraints, respectively. Sections 7.6 and 7.7 compare the multiple-project multiple-option model and the traditional model with

respect to competition and usage aspects, respectively. Finally, concluding remarks are provided in Chapter 7.8.

7.2 Generation Planning Process Based on the Real Options Approach vs. the Traditional Approach

Figure 7.1 depicts the generation planning process based on the real options approach. Figure 7.1 is explained by the following steps focusing on the multiple-project multiple-option model in Chapter 5.

- Step 1. We focus on strategic decisions for a utility. These decisions are represented by strategic options such as the options to construct, to defer, and to expand the constructed capacity level.
- Step 2. Since strategic decisions are long-term decisions, we consider the market values of the completed power plants as the underlying assets for the options (see e.g., [48]).
- Step 3. Based on the studies in [7], [48], [49], the movement of the market values of the completed power plants can be modeled as geometric Brownian motion processes.
- Step 4. We construct lattice models to approximate the geometric Brownian motion processes. We note that, for example, Black-Scholes option-pricing model is a closed form solution for the value of a single European call option. However, closed form solutions are rare for most of partial differential equations generated for options.
- Step 5. Since options are the derivative assets of the underlying assets, we derive additional lattices for the options as in the case of the underlying assets in Step 4. In a lattice, the net cash flows representing the payoffs of an option under different states (i.e., nodes in a lattice) can be estimated by using the values of the underlying assets.

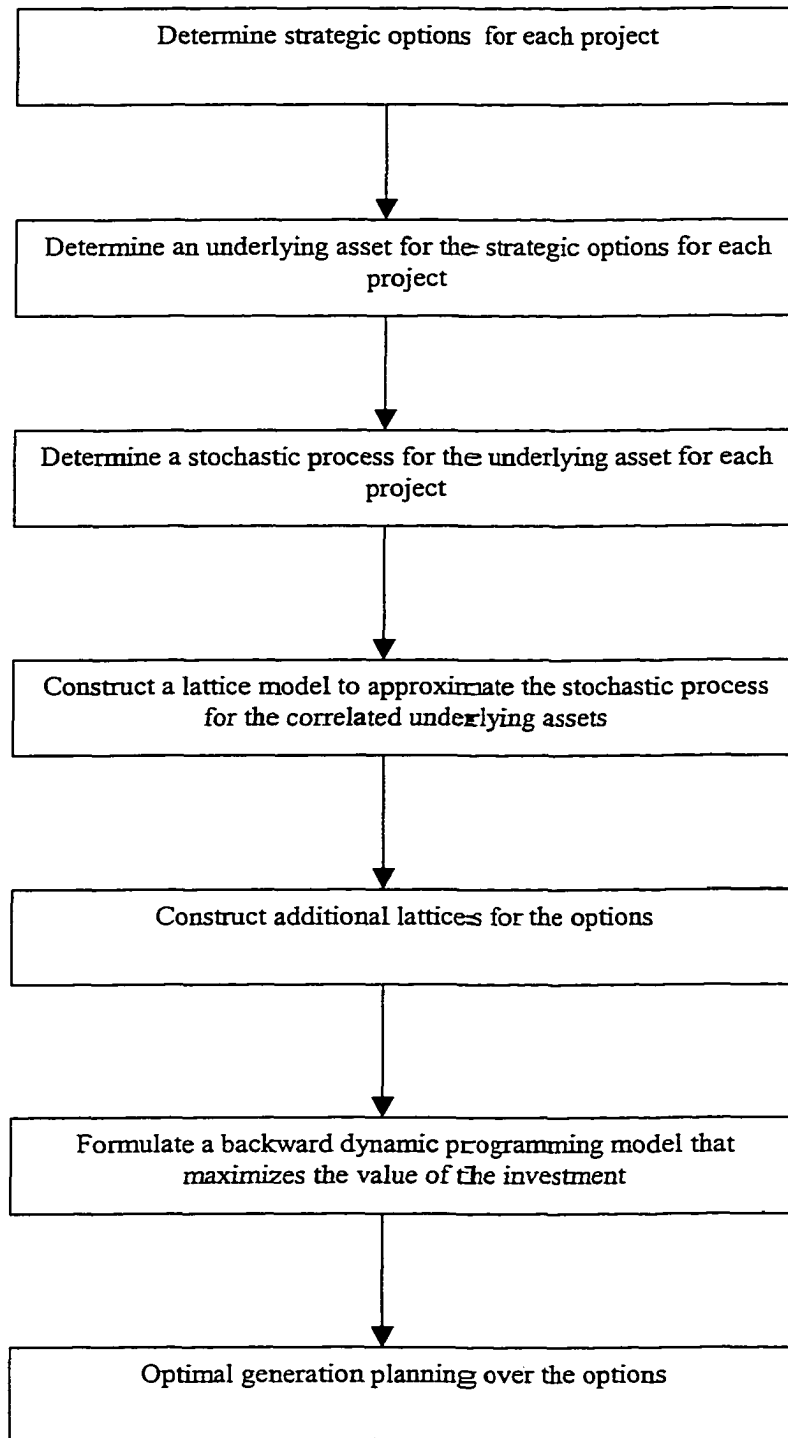


Figure 7.1 Generation planning process based on the real options approach

Step 6. We utilize a backward dynamic programming formulation over the lattice, subject to constraints such as budgets and capacity, to make sequential strategic decisions over options.

Step 7. By maximizing the value of the investment that represents the sum of the accumulated values of options for each project, we will determine the optimal sequential strategic decisions over the options and the corresponding value of the investment.

Thus far, we have examined the generation planning process based on the real options Approach. Let us now proceed to investigate the generation planning process based on the traditional approach (see Figure 7.2). Figure 7.2 is explained by the following steps focusing on the traditional model in Chapter 6.

Step 1. The changes in the electric power industry encourage competition, which in turn encourages more attention to customer satisfaction. We consider the quantified customer outage cost as a good representation for customer satisfaction level. The quantified customer outage cost can be obtained by multiplying customer cost per unsupplied MWh by the expected amount of outage, which is a function of customer demand and available capacity (see e.g., [19], [24]).

Step 2. In the model, we consider both binary decision variables and continuous decision variables (see e.g., [2], [20], [54], [58]). Binary variables are for the project selection purpose. Continuous variables represent the amount of power produced, the portion of construction budget not used, the amount of outside purchase, and the amount of outside sale.

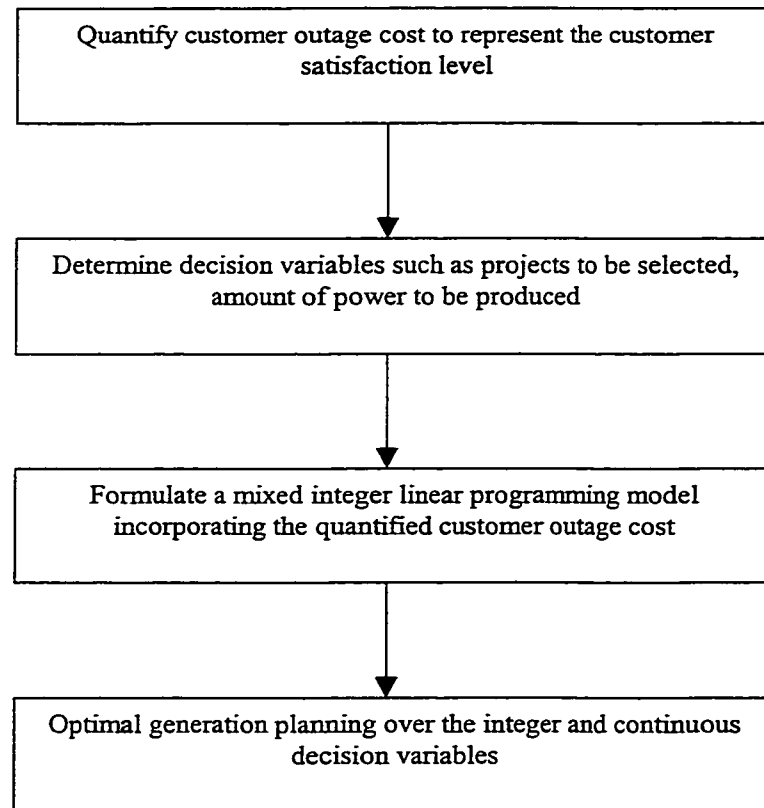


Figure 7.2 Generation planning process based on the traditional approach

Step 3. The objective of our model is to minimize the total cost consisting of discounted fixed, production, utility outage, customer outage, and outside purchase costs minus the discounted revenue from selling power outside (see e.g., [4]). The objective is subject to the constraints of production, purchase, sale, demand, budget, and reliability.

Step 4. The optimal solution of the model will help us make the following decisions: which projects and when the projects should be constructed. For both the selected projects and the existing generation units, the amount of power should be produced, the amount of outside purchase and outside sale should be obtained.

7.3 Comparisons Based on Decision Variables

In the multiple-project multiple-option model, we focus on strategic generation planning (i.e., no tactical generation planning). All possible strategic decisions are represented by strategic options such as construction, expansion, and reduction. The solution of the backward dynamic programming formulation provides the optimal sequential strategic decisions over the planning horizon.

On the other hand, the decision variables in the traditional model are for strategic generation planning and some degrees of tactical generation planning. The binary decision variable will tell us which projects and when the projects should be constructed, which represents strategic generation planning. Furthermore, the continuous decision variables will tell us, for example, how much power should be produced in each period, which represents some degrees of tactical generation planning.

7.4 Comparisons Based on Objective Function

The objective function in the multiple-project multiple-option model represents the value of the investment that represents the sum of the accumulated values of options for each project. By maximizing the objective function for each state in each period subject to constraints, starting from the last period, we will determine the optimal sequential decisions over the options.

On the other hand, the objective function in the traditional model represents the total cost to a utility. This total cost consists of discounted fixed, production, utility outage, customer outage, and outside purchase costs minus the discounted revenue from selling

power outside. By minimizing the total cost to a utility subject to constraints, we will determine the optimal set of projects to be constructed, the optimal amount of power to be produced, the optimal amount of power to be purchased, and the optimal amount of power to be sold.

7.5 Comparisons Based on Constraints

In the multiple-project multiple-option model, there are constraints associated with each state in each period. The constraints may include budgets and capacity requirement constraints. For example, under an unfavorable market condition, a utility may not have enough budgets to construct all proposed projects at the same time. Therefore, there will exist a budget constraint that excludes the option to construct all projects from the set of feasible options.

In the traditional model, there also exist constraints for the available budget for construction and constraints for capacity requirements. Moreover, there exist constraints for some degrees of tactical decisions. For example, the power produced by a generation unit in a period cannot exceed its capacity.

7.6 Comparisons Based on Competition Aspects

The major consequence of competition in the electric power industry is that the price of electric power may be determined by competitive market mechanisms. In such a case, the price of electric power as well as the corresponding rate of return can be much more uncertain than before. In this section, we compare the multiple-project multiple-option model and the traditional model with respect to competition aspects.

Under the new economic uncertainties, the two factors in decision making that have become much more significant are: financial risks and managerial flexibility. In the multiple-project multiple-option model, financial risks are incorporated by risk-neutral probabilities. Furthermore, the possible market conditions are represented by a lattice model. By solving a backward dynamic programming model over the lattice, we determine the optimal strategic decisions under different market conditions. This provides the managerial flexibility.

On the other hand, in the traditional model, we apply an indirect approach to reflect the competitive environments. Under competition, a utility may pay more attention to customer satisfaction. The customer satisfaction level can be represented by customer outage costs. Therefore, we consider customer outage costs as part of the total cost to a utility that should be minimized.

7.7 Comparisons Based on Usage Aspects

With the speed and magnitude of deregulation of generation aspect differing from a state to another state in the U.S., the multiple-project multiple-option model is highly recommendable to the states with the most progress toward deregulation. Maschoff et al. [36] say, “in the past two years, more than 20,000 megawatts of generation assets have been sold, with another 20,000 MW announced. During the next five years, it is expected that 70,000 to 140,000 MW will change hand”. Also, Loehr and Rubin [32] say, “Plans have been announced for construction of about 51,600 MW of merchant generation by the end of 2001”. Under such dramatic environments, the financial risks and the managerial flexibility provided by the multiple-project multiple-option model are vital.

On the other hand, the traditional model is most recommendable to the states with minimum progress toward deregulation. Moreover, our traditional model incorporates customer outage cost which reflects customer satisfaction level. At the 1999 EPRI Summer Seminar, Electric Power Research Institute (EPRI, Palo Alto, California) formal conclusions say, “North America is closer to the edge, in terms of the frequency and duration of severe power outages, than at any time in the last 35 years”. Therefore, a utility that can reduce customer outage costs will more likely obtain better customer satisfaction, which will in turn enhance a utility’s strength in competition.

7.8 Conclusions

In this chapter, first, we explained the generation planning process based on the real options approach and the traditional approach. Then, we compared the multiple-project multiple-option model with the traditional model in terms of decision variables, objectives, constraints, competition, and usage. We note that there are other models based on the real options approach and the traditional approach. Hence, additional comparisons can be made. The models compared in this chapter, however, can be viewed as a first step toward understanding the differences and common aspects of these two approaches.

For future research, it would be interesting to develop a systematic method to determine the most suitable method for generation planning (i.e., the real options approach vs. the traditional approach) based on the magnitude of deregulation.

CHAPTER 8. CONCLUSIONS

8.1 Summary

The electric power industry in many parts of U.S.A. is now undergoing a transition from a regulated monopoly toward an uncertain, competitive market. The magnitude of uncertainties accompanying this transition is truly substantial in generation planning. It is thus highly desirable to help utilities quantitatively and objectively in their decision making process for generation planning.

In this dissertation, to help utilities facing generation planning decisions under market uncertainties, we designed and analyzed a series of mathematical models based on the real options approach.

In view of the real options approach, under market uncertainties, the realization of cash flows often differs from what utilities have expected initially. As new information arrives and uncertainties about market conditions are resolved, the utility management may adjust its strategy to capture future opportunities. Thus, it is highly beneficial for utilities to quantitatively examine various options available in order to exploit the flexibilities that exist and/or may exist in the future.

In Chapter 2, we developed a multiple-project single-option model. In this model, each project has an option to expand its capacity. The Black-Scholes option-pricing formula (table) is used to determine the value of an option. If the project is selected, the project's option becomes available. Thus, the value of a project is represented by the expanded net present value (NPV) which consists of traditional NPV and the value of the option. Then, we

determined the optimal set of projects by maximizing the expanded NPV of selected projects subject to budget constraints and option availability constraints.

The multiple-project single-option model would be useful when there is essentially one dominant option for each project and the projects are independent. However, when there are multiple options for a project, this model may no longer be applicable. For such cases, in Chapter 3, we developed a single-project multiple-option model. In this model, a generation planning project represents a sequence of options. For example, during the construction stage, a utility has the option to construct a power plant or the option to defer the construction. During operating stage, a utility has the option to expand the constructed capacity level or sell the power plant at its salvage value. The selection of an option in one period will affect the selections of options in subsequent periods. Therefore, we employed backward dynamic programming over a binomial lattice to determine the optimal options for a project and the corresponding project value.

Moreover, in the single-project multiple-option model, a utility determines which option to be exercised based on market conditions. Market conditions are represented by the market values of a completed power plant. The market value of a completed power plant can be modeled as a geometric Brownian motion (GBM) process. To avoid complex partial differential equations that may not have analytic solutions, we utilized a binomial lattice model to approximate the GBM process.

The single-project multiple-option is most relevant if there is essentially one important project to be considered. However, when there are multiple projects to be considered, this model may no longer be applicable. We discussed real options models for independent projects in Chapter 2. Let us now proceed to discuss real options models for

correlated projects. In Chapter 4, we developed a two-project multiple-option model. It is assumed that the market values of the two completed power plants are correlated. By employing backward dynamic programming over a four-branch lattice that approximates the combination of two correlated GBMs, we determined the optimal options for the two projects and the corresponding value of the investment. Furthermore, based on the assumption that the two completed power plants are correlated, we investigated the roles of the correlation coefficient in the value of an option and decision making.

Next, in Chapter 5, we extended the two-project multiple-option model to a multiple-project multiple-option model. In this model, the market values of the multiple completed power plants are assumed to be correlated. First, we developed a multiple-branch lattice model to approximate the combination of multiple correlated GBM processes. Then, as in Chapter 4, we employed backward dynamic programming over the lattice to determine the optimal options for the multiple projects and the corresponding values of the investment. Next, we investigated the roles of the correlation coefficients among projects in decision making and the value of an option.

In addition, we provided a traditional generation planning model in Chapter 6 for comparison purposes to the real options models. In this traditional model, we incorporated the outage costs of customers. Current changes in the electric power industry encourage competition, which in turn encourages more attention to customer satisfaction. By including the customer outage cost, we attempted to take the customer satisfaction level into account. Finally, various comparisons between the real options approach and the traditional approach for generation planning are provided in Chapter 7.

We hope that the series of models as well as the data and illustrative numerical examples in this dissertation will help utilities to effectively and efficiently decide and manage generation planning projects with market risks.

8.2 Future Research

There are various worthy extensions possible based on this dissertation. Three possible areas of extensions discussed here are: strategic vs. tactical options, transmission planning, other stochastic models, and other approximation models.

8.2.1 *Strategic vs. Tactical Options*

One extension will be the inclusion of tactical decisions in generation planning. In this dissertation, we focused on strategic decisions such as the options to construct and expand. However, in addition to strategic decisions, a utility also makes tactical decisions such as the options to turn on or turn off a generation unit based on a short-term observation on the spot market.

Strategic options usually involve enormous capital investments. Therefore, it is reasonable to consider the market value of a completed power plant as the underlying asset for strategic options. Based on a relatively long-term observation on the value of a completed power plant, a utility makes strategic decisions. On the other hand, tactical decisions are usually determined based on a relatively short-term observation on the electric power spot market. Therefore, electric power price and fuel price may be the appropriate underlying assets for tactical options.

It is likely that both electric power price and fuel price follow a process other than GBM process. Then, the problem becomes how to consider both strategic and tactical

options in a single model when the two types of options evolve according to different stochastic processes. Furthermore, if the single model can be developed, it will be interesting to investigate the impacts of such a combination on decision making in generation planning.

Another interesting problem will be the reconciliation of the stochastic process for the underlying asset for tactical options and the stochastic process for the underlying asset for strategic options. Since it is likely that tactical decisions will have accumulated impacts on strategic decisions, the accumulated value of tactical options during a period may provide a valuable information for making strategic decisions in a later period. Therefore, it would be worthwhile to investigate the relationship of the stochastic process for the underlying asset for tactical options and for strategic options.

8.2.2 Transmission Planning

In this dissertation, we focused on the developments of project planning and management models with risks for generation planning. However, transmission planning is highly related to generation planning and thus cannot be immune to the economic uncertainties existing in the current electric power industry. Therefore, it is also desirable to apply our project planning and management models with risks to transmission planning as well.

For example, the value of a transmission network is also highly stochastic depending on the transmission volume and time. Given the stochastic market value, the construction process of the transmission network may benefit from a real options approach.

Finally, another extension would be the integrated planning of generation and transmission. Traditional generation planning models have considered both generation planning and transmission planning in a single large-scale mathematical model, because they

are very related. Therefore, it will be worthwhile to consider generation planning and transmission planning in a single management model.

8.2.3 *Other Stochastic Models*

In Subsection 8.2.1, we discussed the strategic vs. tactical options and explained that, for strategic options, the value of a completed power plant would be appropriate as the underlying asset. At the same time, for tactical options, the electricity price would be appropriate as the underlying asset.

We note that the price of electricity, which is stochastic in an uncertain market, can be utilized to make strategic, static (cf. dynamic as in real options models) decisions on generation planning. For example, a mean-variance approach (see Appendix B) can be utilized.

In the near future, it would be interesting to compare and contrast the static mean-variance approach and the dynamic real options approach to derive managerial insights for generation planning.

8.2.4 *Other Approximation Models*

In Section 5.4, we reviewed several different ways to obtain the probabilities for branches of the lattice. One interesting problem is the comparisons of the advantages and disadvantages of each way quantitatively.

In addition, lattice models are not the only methods to approximate the GBM process. Hence, another interesting problem would be the comparisons of the lattice models with numerical methods such as Monte Carlo simulation.

APPENDIX A. RISK-NEUTRAL PROBABILITY

To estimate the value of an option, we can form a replicating portfolio composing of the underlying asset and a risk-free asset. To make this clear, let us consider a single-period binomial lattice model. E_0 represents the value of an option in period 0. E_1^1 and E_1^2 represent the outcomes of the option at state 1 (up) and at state 2 (down) in period 1, respectively. E_1^1 and E_1^2 are known. We want to value E_0 based on E_1^1 and E_1^2 .

Let us purchase x dollars worth of X (e.g., x dollars worth of the stock of the existing generation unit) and y dollars worth of the risk-free asset in period 0 that will duplicate the values of E_1^1 and E_1^2 in period 1. Let r denote risk-free rate of return. In the next period, $x + y$ will be worth either $ux + (1 + r)y$ if market conditions go up or $dx + (1 + r)y$ if market conditions go down. Therefore, we have

$$\begin{aligned} ux + (1 + r)y &= E_1^1 \\ dx + (1 + r)y &= E_1^2 \end{aligned}$$

We have two unknowns, x and y , and two equations. We can solve x and y as

$$\begin{aligned} x &= \frac{E_1^1 - E_1^2}{u - d} \\ y &= \frac{uE_1^2 - dE_1^1}{(u - d)(1 + r)} \end{aligned}$$

Therefore, the value of the portfolio in period 0 is

$$\begin{aligned} x + y &= \frac{E_1^1 - E_1^2}{u - d} + \frac{uE_1^2 - dE_1^1}{(u - d)(1 + r)} \\ &= \frac{1}{1 + r} \left(\frac{1 + r - d}{u - d} E_1^1 + \frac{u - 1 - r}{u - d} E_1^2 \right) \end{aligned}$$

By the no-arbitrage principle, $x + y$ must be equal to E_0 . Let $p = \frac{1+r-d}{u-d}$, then the value of the option in period 0 is

$$E_0 = \frac{1}{1+r}(pE_1^1 + (1-p)E_1^2)$$

In the above equation, we discount the expected value of the option in period 1 at risk-free rate of return to obtain the value of the option in period 0. Therefore, p is called risk-neutral probability.

APPENDIX B. GENERATION PLANNING VIA A MEAN-VARIANCE APPROACH: PROFIT VS. RISK

B.1 Introduction

Traditional generation planning models have rarely emphasized the stochastic price of electricity. Therefore, the traditional models may not fully address the risk in capital investment decision processes under increased financial uncertainties and economic competition. In this appendix, we attempt to fully reflect the financial risk aspect in capital investment decision processes by designing and analyzing a particular type of generation planning models based on the mean-variance approach (see e.g., [55]).

In order to quantitatively incorporate the risk in capital investment decision processes, we need to first define risk mathematically. We note that the level of risk may be represented by the variance of profit (see e.g., [28]) or by the standard deviation of profit (see e.g., [30]). In this appendix, we will employ both representations in the capital investment decision processes. Given these mathematical definitions of risk, the rest of model development is as follows:

In this appendix, the price of electricity is assumed to be a discrete random variable. Employing this random variable, we then obtain the conditions under which a generation unit may be temporarily shut down based on profitability. Next, we derive the mathematical expressions for the mean of the total profit and the variance of the total profit. Finally, we formulate a mean-variance model that minimizes the variance of the total profit subject to the minimum acceptable total profit requirement.

We then proceed to point out a key limitation of the mean-variance model, and propose a weighted mean minus standard deviation model that alleviates this limitation. In this model, an objective function for maximizing the profit while minimizing the variance is formulated. To assign priority weights on the profit relative to the variance, we employ the analytical hierarchy process (AHP). Via numerical examples, we briefly compare the mean-variance model with the weighted mean minus standard deviation model.

The organization of this appendix is as follows. In Section B.2, we present the mean-variance model. In Section B.3, we formulate a weighted mean minus standard deviation model. Section B.4 provides the comparison of the mean-variance model and the weighted mean minus standard deviation model. Concluding remarks and comments on future research are provided in Section B.5. Finally, we note that, throughout this appendix, we will use the terms of “generation unit” and “project” interchangeably.

B.2 A Mean-Variance Model

In this section, we first present the conditions under which a generation unit may be temporarily shut down based on profitability. Employing these conditions, next, we formulate mathematical expressions for the expected (i.e., mean) total profit and the variance of the total profit for generation expansion planning projects. Finally, a mean-variance model is established.

B.2.1 Shutdown and Operating Conditions

We first define the notations used in this section.

Notation:

j = index of generation units, $j=1, \dots, J$

t = index of periods, $t=1, \dots, T$

P_{jt} = discounted per unit selling price of the energy from generation unit j in period t (\$/kWh). It is assumed to be a discrete random variable that has a price level of a_{mjt} with probability α_{mjt} for $m=1, \dots, M_{jt}$. P_{jt} 's are assumed to be independent over j and t . M_{jt} represents the total number of possible prices (states) based on general economic conditions, levels of competition, etc. for generation unit j in period t .

$d_{jt}(P_{jt})$ = energy sold by generation unit j in period t , a function of P_{jt} (kWh)

c_{jt} = discounted per unit variable cost of the energy of generation unit j in period t (\$/kWh)

f_{jt} = discounted fixed cost of operation for generation unit j in period t (\$)

dp_{jt} = discounted depreciation expense for generation unit j in period t (\$)

db_{jt} = discounted annual loan repayment of debt for generation unit j in period t (\$)

As the electric power industry moves away from a regulated monopoly toward competition, the price of electricity can not be deterministically decided by a utility. Instead, the price will be determined by market competition. For the utility, therefore, the price is a random variable with several possible prices (a_{mjt} 's) and their corresponding probabilities (α_{mjt} 's) depending upon the general economic conditions, levels of competition, etc.

From these definitions, if generation unit j operates in period t , the revenue is given by $P_{jt} * d_{jt}(P_{jt})$ and the cost is given by $c_{jt} * d_{jt}(P_{jt}) + f_{jt} + dp_{jt} + db_{jt}$. Therefore, the corresponding profit is given by $P_{jt} * d_{jt}(P_{jt}) - c_{jt} * d_{jt}(P_{jt}) - f_{jt} - dp_{jt} - db_{jt}$.

On the other hand, if the price gets too low, the utility may temporarily shut down a generation unit. In this section, we will assume that each temporary shutdown lasts one period. If generation unit j gets temporarily shut down during period t , the revenue is given by zero and the cost is given by $dp_{jt} + db_{jt}$. Therefore, the corresponding profit is given by $-dp_{jt} - db_{jt}$.

Now, we assume that the utility will temporarily shut down a generation unit for one period based on the level of profit. We define the desired minimum profit to be β_{jt} for generation unit j in period t . Therefore, if the profit is greater than or equal to β_{jt} , then generation unit j will operate in period t . On the other hand, if the profit is less than β_{jt} , then generation unit j will be shut down in period t .

We note that β_{jt} can be negative (i.e., even if the profit is negative, the utility may still operate, especially when the negative amount is not substantial). Now, we define the profit from generation unit j in period t under shutdown and operating conditions by

$$\begin{aligned} \pi_{jt}(P_{jt}) &= P_{jt} * d_{jt}(P_{jt}) - c_{jt} * d_{jt}(P_{jt}) - f_{jt} - dp_{jt} - db_{jt} \\ &\quad \text{if } \pi_{jt}(P_{jt}) = P_{jt} * d_{jt}(P_{jt}) - c_{jt} * d_{jt}(P_{jt}) - f_{jt} - dp_{jt} - db_{jt} \geq \beta_{jt} \\ &= -dp_{jt} - db_{jt} \\ &\quad \text{if } \pi_{jt}(P_{jt}) = P_{jt} * d_{jt}(P_{jt}) - c_{jt} * d_{jt}(P_{jt}) - f_{jt} - dp_{jt} - db_{jt} < \beta_{jt} \end{aligned} \quad (\text{B.1})$$

B.2.2 Mean and Variance of the Total Profit

Based on equation (B.1), an expression for the expected profit for generation unit j is given by

$$\begin{aligned}
E[\pi_j] &= \sum_{t=1}^T E[\pi_{jt}(P_{jt})] \\
&= \sum_{t=1}^T (\text{Prob}\{\text{generation unit } j \text{ is operating in period } t\} * \\
&\quad \text{Profit of generation unit } j \text{ in period } t \mid \text{operating} \\
&\quad + \text{Prob}\{\text{generating unit } j \text{ is temporarily shut down in period } t\} * \\
&\quad \text{Profit of generation unit } j \text{ in period } t \mid \text{shut down})
\end{aligned} \tag{B.2}$$

To develop the expected total profit (mean of the total profit) for generation expansion planning projects, first let A_{jt} be the set of indices of prices at which generation unit j will operate in period t . A_{jt}^C is the complement of A_{jt} , and it denotes the set of indices of prices at which generation unit j will be shut down. Namely,

$$\begin{aligned}
A_{jt} &= \{m \mid a_{mjt} * d_{jt}(a_{mjt}) - c_{jt} * d_{jt}(a_{mjt}) - f_{jt} - dp_{jt} - db_{jt} \geq \beta_{jt}\} \\
A_{jt}^C &= \{m \mid a_{mjt} * d_{jt}(a_{mjt}) - c_{jt} * d_{jt}(a_{mjt}) - f_{jt} - dp_{jt} - db_{jt} < \beta_{jt}\}
\end{aligned}$$

We let the binary variables x_j be 1 if generation expansion planning project j is selected and 0 otherwise. We also assume that all selected projects are operational starting from period 1. Based on equation (B.2), we have the following mathematical expression for the expected total profit (mean of the total profit):

$$\begin{aligned}
E[\pi] &= \sum_{j=1}^J x_j * \sum_{t=1}^T E[\pi_{jt}(P_{jt})] \\
&= \sum_{j=1}^J x_j * \sum_{t=1}^T \left\{ \sum_{m \in A_{jt}} \alpha_{mjt} * (a_{mjt} * d_{jt}(a_{mjt}) - c_{jt} * d_{jt}(a_{mjt}) - f_{jt} - dp_{jt} - db_{jt}) \right. \\
&\quad \left. + \sum_{m \in A_{jt}^C} \alpha_{mjt} * (-dp_{jt} - db_{jt}) \right\}
\end{aligned}$$

Since we assume that the price of electricity is a random variable, the profit is also a random variable. Because the profit will have variance that represents the level of risk, it is

highly desirable to consider the variance of the total profit in any capital budgeting decision process. We have the following mathematical expression for the variance of the total profit:

$$\begin{aligned}
V[\pi] &= V\left[\sum_{j=1}^J x_j * \sum_{t=1}^T \pi_{jt}(P_{jt})\right] \\
&= \sum_{j=1}^J x_j^2 * \sum_{t=1}^T \{E[\pi_{jt}(P_{jt})]^2 - [E(\pi_{jt}(P_{jt}))]^2\} \\
&= \sum_{j=1}^J x_j^2 * \sum_{t=1}^T \left\{ \sum_{m \in A_{jt}} \alpha_{mjt} * (a_{mjt} * d_{jt}(a_{mjt}) - c_{jt} * d_{jt}(a_{mjt}) - f_{jt} - dp_{jt} - db_{jt})^2 \right. \\
&\quad + \sum_{m \in A_{jt}^c} \alpha_{mjt} * (-dp_{jt} - db_{jt})^2 \\
&\quad - \left[\sum_{m \in A_{jt}} \alpha_{mjt} * (a_{mjt} * d_{jt}(a_{mjt}) - c_{jt} * d_{jt}(a_{mjt}) - f_{jt} - dp_{jt} - db_{jt}) \right. \\
&\quad \left. \left. + \sum_{m \in A_{jt}^c} \alpha_{mjt} * (-dp_{jt} - db_{jt}) \right]^2 \right\}
\end{aligned}$$

B.2.3 A Mean-Variance Model

In this subsection, we present the mean-variance model, which is conceptually similar to the mean-variance portfolio selection in Markowitz [35]. In this model, we minimize the variance of the total profit subject to minimum acceptable total profit requirement.

Let F be the minimum acceptable level of profit requirement. Then, a mean-variance model is formulated as follows:

$$\text{Min } V[\pi]$$

Subject to

$$E[\pi] \geq F$$

where the decision variables are over which generation units will be selected.

B.2.4 A Numerical Example

In this example, there are three generation expansion planning projects and the planning horizon is one period. The hypothetical data are listed below.

Data for project 1:

$$d_{11}(P_{11}) = 1,000 - 5 * P_{11}$$

$$P_{11} = \left\{ \begin{array}{l} a_{111} = 70 \text{ with probability } \alpha_{111} = 0.2, d_{11}(70) = 650 \\ a_{211} = 80 \text{ with probability } \alpha_{211} = 0.6, d_{11}(80) = 600 \\ a_{311} = 90 \text{ with probability } \alpha_{311} = 0.2, d_{11}(90) = 550 \end{array} \right\}$$

$$c_{11} = 30; f_{11} = 2,000; dp_{11} = 2,000; db_{11} = 2,000; \beta_{11} = -1,200$$

Data for project 2:

$$d_{21}(P_{21}) = 800 - 2 * P_{21}$$

$$P_{21} = \left\{ \begin{array}{l} a_{121} = 60 \text{ with probability } \alpha_{121} = 0.2, d_{21}(60) = 680 \\ a_{221} = 80 \text{ with probability } \alpha_{221} = 0.4, d_{21}(80) = 640 \\ a_{321} = 100 \text{ with probability } \alpha_{321} = 0.4, d_{21}(100) = 600 \end{array} \right\}$$

$$c_{21} = 50; f_{21} = 1,000; dp_{21} = 2,000; db_{21} = 2,000; \beta_{21} = -800$$

Data for project 3:

$$d_{31}(P_{31}) = 900 - 15 * P_{31}$$

$$P_{31} = \left\{ \begin{array}{l} a_{131} = 40 \text{ with probability } \alpha_{131} = 0.5, d_{31}(40) = 300 \\ a_{231} = 50 \text{ with probability } \alpha_{231} = 0.3, d_{31}(50) = 150 \\ a_{331} = 60 \text{ with probability } \alpha_{331} = 0.2, d_{31}(60) = 0 \end{array} \right\}$$

$$c_{31} = 35; f_{31} = 1,000; dp_{31} = 500; db_{31} = 500; \beta_{31} = -200$$

From these data, we can verify $A_{11} = \{1,2,3\}$, $A_{21} = \{1,2,3\}$, and $A_{31} = \{2\}$ while $A_{11}^C = \phi$, $A_{21}^C = \phi$, and $A_{31}^C = \{1,3\}$. It can be verified that the expected profit for projects 1, 2, and 3 is 24,000; 16,040; and -625, respectively. It can also be verified that the variance of profit for projects 1, 2, and 3 are 4,960,000; 74,022,400; and 328,125, respectively.

Therefore, we have a binary integer programming formulation for the mean-variance model as follows:

$$\begin{array}{l} \text{Min} \\ x_1, x_2, x_3 \end{array} 4960000x_1^2 + 74022400x_2^2 + 328125x_3^2$$

Subject to

$$24000x_1 + 16040x_2 - 625x_3 \geq F$$

We note that the variables x_1^2 , x_2^2 , and x_3^2 can be equivalently represented by x_1 , x_2 , and x_3 , resulting in the same optimal solution. Hence, employing the LINDO software package (LINDO Systems Inc. [29]), when $F = 20,000$, then the optimal solution is $x_1 = 1, x_2 = 0, x_3 = 0$ (i.e., 4,960,000 is the smallest variance among feasible solutions). Hence project 1 is selected. The corresponding profit is 24,000. On the other hand, when $F = 40,000$, then the optimal solution is $x_1 = 1, x_2 = 1, x_3 = 0$. Hence projects 1 and 2 are selected. The corresponding profit and variance are 40,040 and 78,982,400, respectively. We observe that, by doubling the minimum acceptable level of profit requirement from 20,000 to 40,000, the optimal variance has substantially increased from 4,960,000 to 78,982,400.

B.3 A Weighted Mean Minus Standard Deviation Model

The advantage of the mean-variance model is its simplicity. By considering x_1^2 , x_2^2 , and x_3^2 by x_1 , x_2 , and x_3 , the quadratic objective function of the mean-variance model becomes linear and can be easily solved. However, one can observe a key limitation of the mean-variance model in the fixed minimum acceptable level of profit requirement.

This limitation can be explained in the following hypothetical example. Suppose there are two feasible solutions. Solution 1 is the optimal solution with the minimum variance with the profit level at exactly F (the minimum acceptable level of profit requirement). Solution 2, on the other hand, has a slightly higher variance with substantially higher profit relative to F . By the design of the mean-variance model, solution 2 will not be selected. This, in practice, however, may not be a reasonable capital budgeting decision process.

To overcome this limitation of the mean-variance model, we propose a weighted mean minus standard deviation model as follows.

For this new model, we observe first that the profit is desirable and should be increased while the variance is undesirable and should be decreased. Hence, a certain type of objective function that maximizes the profit while minimizes the variance would be highly desirable. The key question then becomes what weight to assign on the profit relative to the variance. This weight should directly address the possibility of trade-off between the profit and the variance (i.e., high profit with high variance vs. low profit with low variance).

To assign weights, we first employ the standard deviation to represent the variance. In this way, the terms of the objective function (the profit minus the standard deviation) will have the same unit. Next, for relative (or priority) weights of the profit vs. the standard deviation, we employ the analytic hierarchy process (AHP) as follows.

B.3.1 Priority Weights by AHP

AHP (see e.g., Saaty [43]) is an effective way to determine relative priority weights. For this purpose, we first need to define the goal and the criteria in an AHP diagram. In this AHP diagram, the goal is to determine the priority weights, and the criteria are the standard deviation and mean of total profit. Next, we need to construct a hierarchy based on the

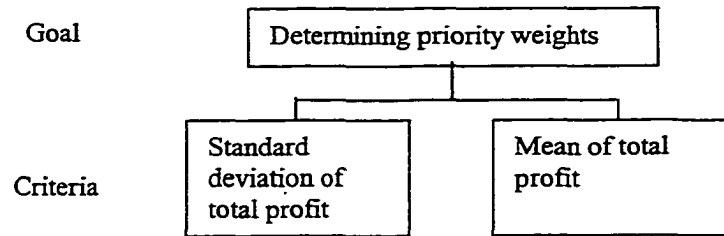


Figure B.1 An AHP diagram for the mean vs. standard deviation

criteria of the standard deviation vs. the mean. The hierarchy for our model is shown in Figure B.1. We note that a hierarchy can be constructed with more criteria and/or subcriteria.

Based on the diagram in Figure B.1, by going through the AHP, we can obtain the priority weights. The actual process of AHP will be explained via a numerical example in the next subsection. We now proceed to the weighted mean minus standard deviation model.

B.3.2 A Weighted Mean Minus Standard Deviation Model

Let w_1 be the priority weight of the mean and w_2 be the priority weight of the standard deviation from the AHP. Then, a weighted mean minus standard deviation model is formulated as follows:

$$\text{Max } w_1 * E[\pi] - w_2 * \sqrt{V[\pi]}$$

where the decision variables are over which generation units will be selected.

B.3.3 A Numerical Example

In this subsection, we illustrate some of the features of our model via a numerical example. First, in order to determine the priority weights, pairwise comparisons are necessary. The pairwise comparisons between the standard deviation and the mean can be organized into a pairwise comparison matrix. The pairwise comparison matrix consists of

e_{ij} 's. e_{ij} is a number representing the level of importance of criterion i relative to criterion j . There are several important characteristics in the pairwise comparison matrix: When compared with itself, each criterion has equal importance. Therefore, the diagonal elements of the pairwise comparison matrix are all equal to 1. Moreover, the lower triangle numbers of the matrix are the reciprocal of the upper triangle numbers (i.e., $e_{ij} = 1/e_{ji}$). All numbers in the matrix are positive.

For example, let us assume that the standard deviation of the total profit is two times as important as the mean of the total profit. Then, a pairwise comparison matrix is formed as shown in Table B.1.

Table B.1 Structure of pairwise comparison matrix

	Standard deviation	Mean
Standard deviation	1	2
Mean	1/2	1

After the pairwise comparison matrix is formed, we can calculate the priority weights of criteria. First, we divide each e_{ij} of the j th column by the sum of e_{ij} 's of the j th column. Then, we add all e_{ij} 's in each resulting row. Finally, the sum is divided by the number of columns. Via this normalization, we obtain the priority weights of criteria (see e.g., Son and Min [46]).

Therefore, from Table B.1, we first divide 1 and 1/2 of the first column by 3/2, which is the sum of 1 and 1/2. We obtain 2/3 and 1/3 in the first column of Table B.2. Similarly, we divide 2 and 1 of the second column in Table B.1 by 3, which is the sum of 2 and 1. We obtain 2/3 and 1/3 in the second column of Table B.2. Next, we calculate the sum of each

row in Table B.2. The sum of the first row in Table B.2 is equal to $4/3$ (1.333). The sum of the second row in Table B.2 is equal to $2/3$ (0.667). Finally, we divide 1.333 by 2, which is the number of columns, and obtain 0.666. We note that 0.666 is the priority weight for the standard deviation of the total profit (i.e., $w_2 = 0.666$). Similarly, we divide 0.667 by 2 and obtain 0.334. We note that 0.334 is the priority weight for the mean of the total profit (i.e., $w_1 = 0.334$).

Table B.2 Priority weights of criteria

	Standard dev.	Mean	Row sum	Priority weights
Standard dev.	2/3	2/3	1.333	0.666
Mean	1/3	1/3	0.667	0.334

We note that, in this section, we do not address the issue of consistency in comparison matrices because the data are hypothetical. In practice, with real data, one needs to address this issue (see e.g., Saaty [43]).

Given these priority weights, recall that the mean and variance of each project, $j = 1, 2, 3$, are provided in Subsection 2.2.4. Hence, the corresponding weighted mean minus standard deviation model is as follows:

$$\begin{aligned} \underset{x_1, x_2, x_3}{Max} \quad & 0.334 * (24000x_1 + 16040x_2 - 625x_3) \\ & - 0.666 * \sqrt{4960000x_1^2 + 74022400x_2^2 + 328125x_3^2} \end{aligned}$$

where x_1, x_2, x_3 are binary integers.

This binary nonlinear integer programming problem is solved by enumeration, and the corresponding global optimal solution is: $x_1 = 1$, $x_2 = 1$, and $x_3 = 0$. i.e., projects 1 and 2 are selected. The corresponding objective function value is 7454.48. We note that LINGO

software package (LINDO Systems Inc. [29]) can solve binary nonlinear integer programming problems. However, by LINGO, only a local optimum can be identified.

B.4 Comparison of Results

The optimal solution for the mean-variance model when $F = 20,000$ is $x_1 = 1, x_2 = 0, x_3 = 0$. The corresponding mean and variance of the total profit are 24,000 and 4,960,000, respectively. We do note that there may be feasible solutions that have substantially higher profits and slightly higher variances than this optimal solution (which is a key limitation of the mean-variance model).

The optimal solution for the weighted mean minus standard deviation model is $x_1 = 1, x_2 = 1, x_3 = 0$. The corresponding mean and variance of the total profit are 40,040 and 78,982,400, respectively. We first observe that both mean and standard deviation are considered in the objective. Hence, it is now possible that the feasible solution that has substantially higher profit and slightly higher variance of the previous case becomes the optimal solution. We also observe that the priority weights would depend on a particular decision maker's degree of risk-aversion. i.e., if the decision maker is more (less) risk-averse, then the priority weight for the standard deviation will be higher (lower).

B.5 Concluding Remarks

In this appendix, we first presented the conditions under which a generation unit may be temporarily shut down based on profitability. From these conditions, we presented the mathematical expressions for both mean and variance of the total profit. Then, we formulated a mean-variance model. From this model, we selected the projects that would

minimize the variance of the total profit subject to the minimum acceptable total profit requirement.

Next, we introduced how to obtain priority weights for the mean and standard deviation of the total profit via the AHP. By assigning these priority weights to the mean and the standard deviation, we formulated a weighted mean minus standard deviation model. Illustrative numerical examples were provided for both mean-variance model and weighted mean minus standard deviation model. Finally, comparison of the results of the numerical examples was presented.

For future research, it would be worthwhile to examine various economic and financial conditions under which the mean-variance model (or the weighted mean minus standard deviation model) is more appropriate.

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